

# A Topological Theory of the Physical Vacuum

**R. M. Kiehn**

Emeritus Professor of Physics

University of Houston

<http://www.cartan.pair.com>

© Copyright CSDC INC. 2005

December 23, 2005

**Abstract**

**DRAFT3**

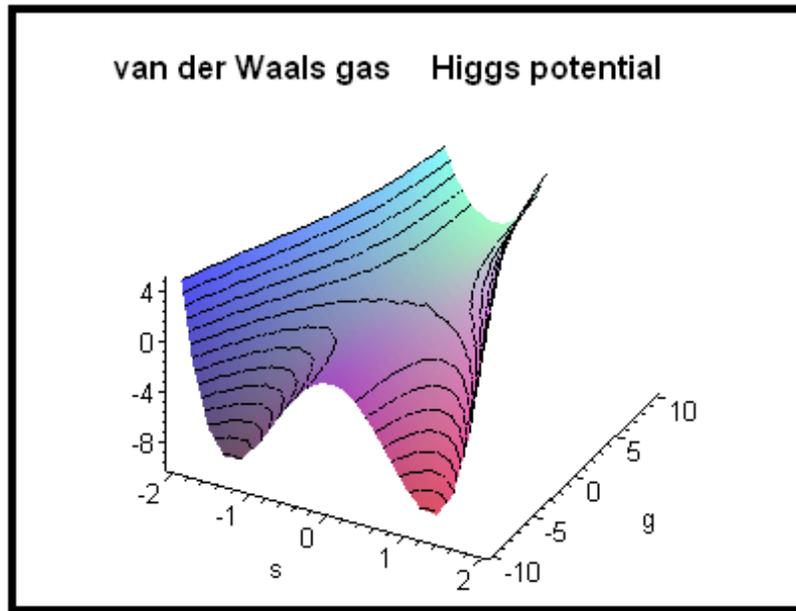
Shipov's concept of a "Physical Vacuum" is extended to include sets that do not require the global features of "Absolute Parallelism". The theory is developed in terms of vectors and matrices of exterior differential forms, which permit the topological coherent structures of fields and particles that make up a "Physical Vacuum", as well as their topological fluctuations, to be readily evaluated in terms of a more or less "universal" theory. The sole requirement is that the "Physical Vacuum" be defined as a vector space of infinitesimal linear neighborhoods whose points admit a vector Basis Frame, as a matrix of  $C^2$  functions with non-zero determinant, to be defined over the domain of the "Physical Vacuum". The topological universality of a Basis Frame over infinitesimal neighborhoods can be specialized by particular choices of a subgroup structure. Such specializations appear to include the field structures of all four forces, from gravity fields to Yang Mills fields.

## 1 Preface

In 1993-1998, Gennady Shipov [51] presented his pioneering concept of a "Physical Vacuum" as a space of "Absolute Parallelism".

The point of departure in this article consists of three parts:

I Shipov's vision that a "Physical Vacuum" is a space of Absolute Parallelism is extended to include a larger set of admissible systems. The larger set is based on the sole requirement that *infinitesimal* neighborhoods of a "Physical Vacuum" are linearly connected as a vector space. The additional (*global*) constraint of "Absolute Parallelism" is not utilized (necessarily). The sole requirement implies that the points  $\{y^a\}$  of the "Physical Vacuum" support a matrix of C2 functions, with a non-zero determinant. This matrix of functions is defined as a Basis Frame,  $[\mathbb{B}(y^a)]$ , for the "Physical Vacuum", and represents the vector space properties of infinitesimal neighborhoods. When the determinant of the Basis Frame is not zero, the domain of definition is 4 dimensional, and the matrix of functions  $[\mathbb{B}]$  has a Cayley-Hamilton polynomial of 4th degree which defines a family of implicit surfaces in a 4 dimensional space. The Cayley-Hamilton polynomial defines a thermodynamic Phase function (see Section 3.4, [54]). The polynomial also has a universal envelope which can be expressed in terms of a quartic polynomial, which I have defined as the Higgs surface (Thermodynamic Phase) function,  $k(s, g) = 0$ , comparable to a universal van der Waals gas.

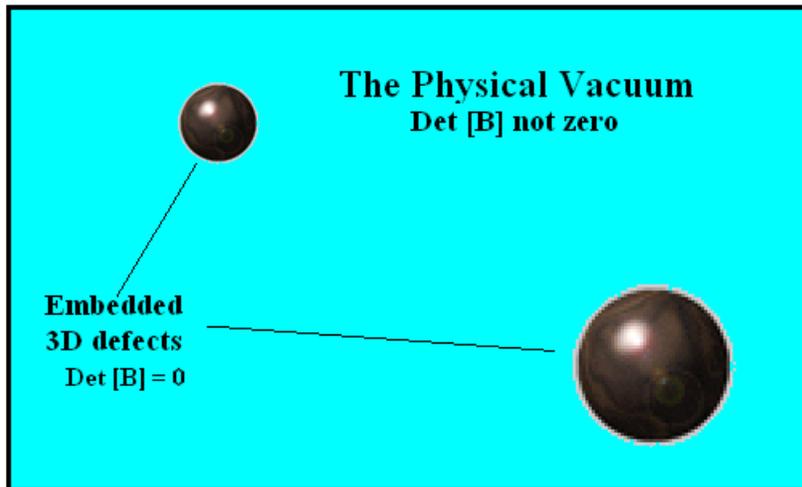


The vertical axis is the Higgs function,  $k$ .  
**Singularities in  $[\mathbb{B}]$  occur when  $k = 0$  or  $\partial k / \partial s = 0$ .**  
**The critical point is where  $k = s = g = 0$ .**

It should be noted that the Higgs component,  $k$ , is related to the determinant of the Basis Frame, and is an artifact of 4 dimensions. The  $s$  component is related to abstract molar

(mass) density in a thermodynamic interpretation, and  $g$  has features of a temperature. Details are to be found in [54].

II If the Frame Matrix  $[\mathbb{B}]$  is singular, then 1 or more of its eigenvalues is zero. When  $s = 0$ , or  $\partial k / \partial s = 0$ , singularities occur such the  $[\mathbb{B}]^{-1}$  is not defined. These objects may be viewed as topological defects of 3 dimensions or less embedded in the domain of 4 dimensional "Physical Vacuum". They can be thought of as condensates. The major theme of this article examines the features of the "Physical Vacuum", which is the domain which is free of singularities of the type  $\det[\mathbb{B}] = 0$ . The Basis Frame Matrix  $[\mathbb{B}]$  will be assumed to consist of C2 functions, but only C1 is required for deriving a linear connection that defines infinitesimal differential closure. If the functions are not C2, singularities can occur in second order terms, such as curvatures (and accelerations).



The 4D Physical Vacuum with 3D topological defects

1. Although more complicated, the singular sets admit analysis, for example, in terms of propagating discontinuities and topologically quantized period integrals [13]. These topics will be considered in a later article.

III It is recognized that topological coherent structures (fields, and particles, along with fluctuations) in a "Physical Vacuum" can be put into correspondence with the concepts of topological thermodynamics based on continuous topological evolution [27] [54]. Perhaps surprising to many, topology can change continuously in terms of processes that are not diffeomorphic. The mathematics will utilize vector and matrix arrays of Cartan's exterior differential forms. Such arrays, whose elements are

exterior differential p-forms, are automatically covariant with respect to diffeomorphisms. Topological properties are inherent in the differential systems defined by such structures. The topological properties of such mathematical structures and their evolutionary dynamics can be described in terms of the Lie directional differential (which admits topological change), not the infamous "Covariant" differential of tensor analysis defined in terms of a linear connection (which does not admit topological change consistent with a constraint of diffeomorphism).

## 2 The Fundamentals of the Theory in 4D

1. Assume the existence of a matrix array of 0-forms (functions),  $[\mathbb{B}]$ , on a 4D variety of points  $\{y^a\}$ . The domain for which the determinant of  $[\mathbb{B}]$  is not zero will be defined as a Physical Vacuum, and in such regions, there exists an an inverse Frame,  $[\mathbb{B}]^{-1}$ . The compliment of the Physical Vacuum is defined as the singular domain, and is the realm of topologically coherent defect structures (such as particles) which will be described later.
2. The Basis Frame  $[\mathbb{B}(y)]$  can be used to map vector arrays of exact differentials  $|dy^a\rangle$  into nearby vector arrays of 1-forms  $|\sigma^a\rangle$  :

$$[\mathbb{B}(y)] \circ |dy^a\rangle \Rightarrow |\sigma^a\rangle. \quad (1)$$

Note that  $|\sigma^a\rangle$  is a vector array of 1-forms, and is not the same as a "coframe" matrix of 1-forms which makes up a cornerstone of Metric-Affine Gravity theories [1]. The 1-forms  $|\sigma^a\rangle$  can be exact, closed, integrable, or non integrable. These are topological properties of the Pfaff Topological dimension (Class) of each of the 1-forms,  $\sigma^a$ .

3. From the identity  $[\mathbb{B}] \circ [\mathbb{B}]^{-1} = [\mathbb{I}]$ , use exterior differentiation to derive the (right) Cartan Connection  $[\mathbb{C}]$  as a matrix of 1-forms.

$$\text{Right} \quad : \quad \text{Cartan Connection } [\mathbb{C}] \quad (2)$$

$$d[\mathbb{B}] = [\mathbb{B}] \circ [\mathbb{C}], \quad (3)$$

$$[\mathbb{C}] = -d[\mathbb{B}]^{-1} \circ [\mathbb{B}] \quad (4)$$

$$= +[\mathbb{B}]^{-1} \circ d[\mathbb{B}]. \quad (5)$$

4. It is also possible to define a left Cartan Connection as a matrix of 1-forms,  $[\Delta]$ ,

$$\text{Left} \quad : \quad \text{Cartan Connection } [\Delta] \quad (6)$$

$$d[\mathbb{B}] = [\Delta] \circ [\mathbb{B}], \quad (7)$$

$$[\Delta] = -[\mathbb{B}] \circ d[\mathbb{B}]^{-1} \quad (8)$$

$$= +d[\mathbb{B}] \circ [\mathbb{B}]^{-1}. \quad (9)$$

The coefficients that make up the matrix of 1-forms,  $[\Delta]$ , have been called the Weitzenboch connection.

5. The Right and Left Cartan connections are not (usually) identical. They are equivalent in terms of the similarity transformation:

$$[\mathbb{C}] = [\mathbb{B}]^{-1} \circ [\Delta] \circ [\mathbb{B}], \quad (10)$$

Also note that inverse matrix also enjoys differential closure properties.

$$d[\mathbb{B}]^{-1} = [\mathbb{B}]^{-1} \circ [-\Delta], \quad (11)$$

$$= [-\mathbb{C}] \circ [\mathbb{B}]^{-1}. \quad (12)$$

6. If the 1-forms  $|\sigma^a\rangle$  were written to include a constant factor of physical dimensions,  $\hbar/e$ , the resulting 1-forms are formally equivalent to a set of electromagnetic 1-forms of Action (the vector potentials),  $|A^a\rangle$  for each index  $a$ . For purposes of more rapid comprehension - based on the assumption of familiarity with electromagnetic theory - the Basis Frame infinitesimal mapping formula is rewritten as:

$$[\mathbb{B}(y)] \circ |dy^a\rangle \Rightarrow |A^a\rangle. \quad (13)$$

7. Exterior differentiation of the infinitesimal mapping equation generates a vector of (exact) 2-forms  $|F^a\rangle$  which is formally equivalent (for each element,  $a$ ) to the (pair, or even) 2-form of  $\mathbf{E}$ ,  $\mathbf{B}$  field intensities of electromagnetic theory.

$$\text{Vectors of Torsion 2-forms} \quad : \quad (14)$$

$$[\mathbb{B}(y)] \circ [\mathbb{C}] \wedge |dy^a\rangle \Rightarrow |dA^a\rangle = |F^a\rangle, \quad (15)$$

$$[\mathbb{B}(y)] \circ |G\rangle = |F\rangle. \quad (16)$$

$$\text{Field Intensity 2-forms} = |F^a\rangle \quad (17)$$

$$\text{Field Excitation 2-forms} = |G^k\rangle. \quad (18)$$

8. The vector of 2-forms  $|G^k\rangle = [C_a^k] \wedge |dy^a\rangle$  defines precisely the vector of Excitation Torsion 2-forms, relative to the connection  $[C]$ . It is unfortunate that, historically, this anti-symmetry property of the coefficients of the Connection has been described as the vector of "affine"<sup>1</sup> Torsion 2-forms. However, the anti-symmetry concept is described by the same formula,  $|G^k\rangle = [C_a^k] \wedge |dy^a\rangle$ , even though the Basis Frame  $[\mathbb{B}(y)]$  is not an element of the affine group (a transitive group of 13 parameters in 4D, (see p.162 in Turnbull [52]), or one of its subgroups. For example, the Torsion formula holds equally well for the 15 parameter projective group, which is not affine.

$$\text{(Affine) Torsion 2-forms } |G^k\rangle = [C] \wedge |dy^a\rangle, \quad (19)$$

$$\text{Vector of Field Excitation 2-forms} = |G^k\rangle \quad (20)$$

$$= [\mathbb{B}]^{-1} \circ |F^a\rangle, \quad (21)$$

The vector of 2-forms  $|G^k\rangle$  is formally equivalent (for each element  $k$ ) to the (impair, or odd) 2-form (density) of **D, H** field excitations in electromagnetic theory. The matrix  $[\mathbb{B}]^{-1}$  plays the role of the Constitutive map between **E, B** and **D, H**.

$$|G^k\rangle = [\mathbb{B}]^{-1} \circ |F^a\rangle, \quad (22)$$

$$[\mathbb{B}]^{-1} \approx \text{A Constitutive map} \quad (23)$$

Note that none of this development depends upon the specification of a metric.

Although, for simplicity is assumed herein that all functions are C2, note that the definitions of the matrix of 1-forms  $[C]$  and the vector of 2-forms  $|F\rangle$  only require C1 functions.

9. It is possible to use the left Cartan Connection to define another vector of torsion 2-forms. Exterior differentiation of the infinitesimal mapping equation produces the equation yields

$$[\Delta] \wedge ([\mathbb{B}(y)] \circ |dy^a\rangle) \Rightarrow [\Delta] \wedge |\sigma^a\rangle = |d\sigma^a\rangle. \quad (24)$$

The algebraic combination defined below as  $|\Sigma^a\rangle$ , is the Cartan vector of Torsion 2-forms. It is not the same as the vector of excitation Torsion 2-forms defined by  $|G^k\rangle$ .

$$\text{Cartan} \quad : \quad \text{vector of Torsion 2-forms } |\Sigma^a\rangle \quad (25)$$

$$|\Sigma^a\rangle = |d\sigma^a\rangle - [\Delta_b^a] \wedge |\sigma^b\rangle \neq |G^k\rangle. \quad (26)$$

The Cartan vector of Torsion 2-forms,  $|\Sigma^a\rangle$ , is always Zero for "Physical Vacuums", while the vector of excitation Torsion 2-forms,  $|G^k\rangle$  is not.

---

<sup>1</sup>I use the word "affine" in very small print to reinforce that this idea of Torsion is not constrained to be that associated with Affine Matrices. Hopefully, in the future, people will say the "the excitation Torsion coefficients relative to the conformal group, or relative to the affine group with a fixed point, etc.

10. The exterior derivative of the vector of field intensity 2-forms  $|F^a\rangle$  vanishes, resulting in  $a$  sets of partial differential equations of the Maxwell Faraday type:

$$|dF^a\rangle \Rightarrow 0 \quad (27)$$

$$= \text{Maxwell Faraday PDE,s.} \quad (28)$$

The concept requires C2 functions for the potentials  $|A^a\rangle$

11. The exterior derivative of the vector of 2-form excitations  $|G^k\rangle$  produces a vector of 3-forms (formally) equivalent (element by element  $k$ ) to the conserved electromagnetic charge-current density for C2 functions.

$$|dG^k\rangle = |J^k\rangle, \quad (29)$$

$$d|J^k\rangle = 0, \quad (30)$$

$$= \text{Maxwell Ampere PDE,s} \quad (31)$$

12. Construct the (pair - even) Topological Torsion [15] [57] scalar of 3-forms defined as

$$\text{Topological Torsion:} \quad H = \langle A^a | \wedge | F^a \rangle \quad (32)$$

$$= A^1 \wedge F^1 + A^2 \wedge F^2 + A^3 \wedge F^3 + A^4 \wedge F^4. \quad (33)$$

13. Construct the (impair-odd) Topological Spin [16] [57] (pseudo) scalar of 3-forms defined as

$$\text{Topological Spin:} \quad S = \langle A^m | \wedge | G^m \rangle, \quad (34)$$

$$= A^1 \wedge G^1 + A^2 \wedge G^2 + A^3 \wedge G^3 + A^4 \wedge G^4. \quad (35)$$

14. Exterior differentiation of these two 3-forms produces the Poincare invariants

$$dH = \langle F^a | \wedge | F^a \rangle = \sum_a 2(\mathbf{E} \cdot \mathbf{B})^a = \text{Poincare II} \quad (36)$$

$$dS = \langle F^a | \wedge | G^a \rangle - \langle A^a | \wedge | J^a \rangle = \text{Poincare I.} \quad (37)$$

$$= \sum_a [\{(\mathbf{B} \cdot \mathbf{H})^a - (\mathbf{D} \cdot \mathbf{E})^a\} - \{(\mathbf{A} \cdot \mathbf{J})^a - (\rho\phi)^a\}]. \quad (38)$$

The notation in terms of  $\mathbf{E}, \mathbf{B}, \mathbf{D}, \mathbf{H}$  is symbolic, and refers only to the formal equivalence of the formulas to electromagnetic theory. If the Poincare invariants vanish, the closed integrals of the closed 3-forms could become topologically quantized through the concept of deRham period integrals. There are many ways that this result could

happen, due to the fact that there are four (Yang Mills) components to the excitation 2-forms,  $|G^k\rangle$ . Note that if there exists only 1-component of  $|G^k\rangle$ , say  $G^4$ , and it is closed,  $dG^4 = 0$ , then, formally, the integral over a closed 2D integration chain of  $G^4$  defines the Quantized Charge,  $e$ , of classical electromagnetic theory.

The Multi-component Field intensity and Field excitation equations are *extensions* of a Yang Mills theory, as  $\pm$  self duality is not imposed on the system.

15. Use the quadratic congruence (see p. 36 in Turnbull and Aitken [53]) to define the symmetric (metric) qualities of  $[\mathbb{B}]$ :

$$[g] = [\mathbb{B}]^T \circ [\eta] \circ [\mathbb{B}]. \quad (39)$$

The matrix  $[\eta]$  is a (diagonal) Sylvester signature matrix whose elements are  $\pm 1$ . Recall that the congruence transform defines a correlation, where the similarity transform defines a collineation, in projective geometry. However, from the Basis Frame, it is also possible to construct a quadratic form from the law of differential closure. It then follows that for  $d[\eta] = 0$ ,

$$d[g] = [\tilde{\mathcal{C}}_r] \circ [g] + [g] \circ [\mathcal{C}_r], \quad (40)$$

which is the metricity condition for a Physical Vacuum with a right Cartan connection:

$$\textbf{Metricity condition: } d[g] - [\tilde{\mathcal{C}}_r] \circ [g] - [g] \circ [\mathcal{C}_r] \Rightarrow 0. \quad (41)$$

This equation is a exterior differential system, and therefor defines topological properties.

16. Compute the Christoffel connection, and its matrix of 1-forms,  $[\Gamma]$ , from the quadratic "metric" matrix  $[g]$  using the Levi-Civita-Christoffel formulas.

$$\text{Christoffel coefficients} \quad : \quad (42)$$

$$\Gamma_{ac}^b(\xi^c) = g^{be} \{ \partial g_{ce} / \partial \xi^a + \partial g_{ea} / \partial \xi^c - \partial g_{ac} / \partial \xi^e \}, \quad (43)$$

$$\text{as a matrix of 1-forms } [\Gamma] = [\Gamma_{ac}^b dy^c]. \quad (44)$$

17. Decompose the Cartan Connection matrix of 1-forms as follows:

$$[\mathcal{C}] = [\Gamma] + [\mathbb{T}] \quad (45)$$

Note that the topological metricity condition is automatically satisfied for  $[\Gamma]$ , and, as demonstrated above, is satisfied for  $[\mathcal{C}]$ , hence it must also be satisfied for  $[\mathbb{T}]$ .

18. Construct the matrix of Cartan Curvature 2-forms,  $[\Phi]$ , derived from the second exterior differentiation of the Basis Frame,  $dd[\mathbb{B}]$ .

$$dd[\mathbb{B}] = d[\mathbb{B}] \circ [\mathbb{C}] + [\mathbb{B}] \circ d[\mathbb{C}], \quad (46)$$

$$= [\mathbb{B}] \circ \{d[\mathbb{C}] + [\mathbb{C}] \wedge [\mathbb{C}]\}, \quad (47)$$

$$= [\mathbb{B}] \circ [\Phi] \Rightarrow 0, \quad (48)$$

$$\text{Curvature 2-forms } [\Phi] = \{d[\mathbb{C}] + [\mathbb{C}] \wedge [\mathbb{C}]\} \Rightarrow 0 \quad (49)$$

The zero result, based upon the Poincare lemma,  $dd[\mathbb{B}] \Rightarrow 0$ , requires C2 differentiability of the basis functions.

19. Note that exterior differentiation of the Cartan structure matrix of curvature 2-forms is equivalent to the Bianchi identity:

$$[d\Phi] + [d\mathbb{C}] \wedge [\mathbb{C}] - [\mathbb{C}] \wedge [d\mathbb{C}] = \quad (50)$$

$$[d\Phi] + [\Phi] \wedge [\mathbb{C}] - [\mathbb{C}] \wedge [\Phi] \Rightarrow 0 \quad (51)$$

This concept of a Bianchi identity is valid for all forms of the Cartan structure equations. The Bianchi statements are essentially definitions of cohomology, in that the difference between two non-exact p-forms is equal to a perfect differential (an exterior differential system). In this case the Bianchi identity describes the cohomology established by two 3-forms.

$$[J2] - [J1] = [d\Phi], \quad (52)$$

$$\text{where } [J1] = [d\mathbb{C}] \wedge [\mathbb{C}] \quad (53)$$

$$\text{and } [J2] = [\mathbb{C}] \wedge [d\mathbb{C}] \quad (54)$$

20. Substitute  $[\Gamma] + [\mathbb{T}]$  for  $[\mathbb{C}]$  in the definition of the matrix of curvature 2-forms, and recall that for the Physical Vacuum the matrix of curvature 2-forms is zero.

$$[\Phi_{\mathbb{C}}] = \{d[\mathbb{C}] + [\mathbb{C}] \wedge [\mathbb{C}]\} \Rightarrow 0, \quad (55)$$

$$= \{d([\Gamma] + [\mathbb{T}]) + ([\Gamma] + [\mathbb{T}]) \wedge ([\Gamma] + [\mathbb{T}])\} \quad (56)$$

$$= \{d[\Gamma] + [\Gamma] \wedge [\Gamma]\} + \{[\mathbb{T}] \wedge [\Gamma] + [\Gamma] \wedge [\mathbb{T}]\} + \{d[\mathbb{T}] + [\mathbb{T}] \wedge [\mathbb{T}]\}, \quad (57)$$

$$= \{[\Phi_{\Gamma}]\} + \{\text{interaction\_2 - forms}\} + \{[\Phi_{\mathbb{T}}]\} \quad (58)$$

21. Separate the matrices of 2-forms into metric curvature 2-forms,  $\{d[\Gamma] + [\Gamma] \wedge [\Gamma]\} = [\text{Field metric 2 - forms}]$ ,

and the remainder, defined as  $[-(\textit{Inertial} \quad 2 - \textit{forms})]$ . The decomposition leads to the strong equivalence equation,

$$\text{Principle of} \quad : \quad \text{Strong Equivalence} \quad (59)$$

$$[\text{Field curvature 2-forms}] = [\text{Inertial curvature 2-forms}]. \quad (60)$$

22. Consider the Lie differential with respect to a direction field  $V$ , operating on the formula for differential closure

$$L_{(V)}([\mathbb{B}(y)] \circ |dy^a\rangle) = L_{(V)}(|A^a\rangle) = i(V)d|A^a\rangle + d(i(V)|A^a\rangle) \quad (61)$$

$$= i(V)|F^a\rangle + d(i(V)|A^a\rangle) = \quad (62)$$

$$= |W^a\rangle + d|H^a\rangle. \quad (63)$$

From Koszul's theorem,  $|W^a\rangle = i(V)d|A^a\rangle$  is a covariant differential based on a (abstract) connection (for each  $a$ ). Hence, the difference between the Lie differential and the Covariant differential is the exact term,  $d(i(V)|A^a\rangle)$ :

$$L_{(V)}(|A^a\rangle) - i(V)d|A^a\rangle = d(i(V)|A^a\rangle) = d|H^a\rangle. \quad (64)$$

This equation is another statement of Cohomology, another exterior differential system, where the difference of two non-exact objects is an exact differential.

From the topological formulation of thermodynamics [54] in terms of Cartan's magic formula [2],

$$\text{Cartan's Magic Formula } L_{(\rho\mathbf{V}_4)}A = i(\rho\mathbf{V}_4)dA + d(i(\rho\mathbf{V}_4)A) \quad (65)$$

$$\text{First Law} \quad : \quad W + dU = Q, \quad (66)$$

$$\text{Inexact Heat 1-form } Q = W + dU = L_{(\rho\mathbf{V}_4)}A \quad (67)$$

$$\text{Inexact Work 1-form } W = i(\rho\mathbf{V}_4)dA, \quad (68)$$

$$\text{Internal Energy } U = i(\rho\mathbf{V}_4)A, \quad (69)$$

Now consider particular process paths (defined by the directional field  $\rho\mathbf{V}_4$ ), and deduce that in the direction of the process path

$$i(\rho\mathbf{V}_4)W = 0, \quad (70)$$

$$\text{Work} \quad : \quad \text{is transversal}; \quad (71)$$

$$i(\rho\mathbf{V}_4)Q = i(\rho\mathbf{V}_4)dU \neq 0 \quad (72)$$

$$\text{Heat} \quad : \quad \text{is not transversal}; \quad (73)$$

$$\text{but if } i(\rho\mathbf{V}_4)Q = 0, \quad (74)$$

$$\text{the process} \quad : \quad \text{is adiabatic.} \quad (75)$$

It is the non-adiabatic components of a thermodynamic process that indicate that there is a change of internal energy and hence an inertial force in the direction of a process. This implies that the non-adiabatic processes are inertial effects, and could be related to changes in mass.

Now to paraphrase a statement from Mason and Woodhouse, (see p. 49 [3]) :

**Remark 1** *"... then there is a Higgs field.... which measures the difference between the Covariant differential along  $V$  and the Lie differential along  $V$ ."*

1. It becomes apparent that the

$$|W^a\rangle = \text{Vector of Work 1-forms. (transversal)} \quad (76)$$

$$|H^a\rangle = \text{Higgs potential as vector of 0-forms} \quad (77)$$

$$d|H^a\rangle = \text{Higgs vector of 1-forms.} \quad (78)$$

$$i(V)d|H^a\rangle = \text{vector of longitudinal inertial accelerations} \quad (79)$$

$$= \text{non adiabatic components of a process} \quad (80)$$

The method of the "Physical Vacuum" and its sole assumption leads to inertial properties and the Higgs field, all from a topological perspective and without "quantum" fluctuations.

At this point, there has been no indication that the problem being investigated has anything to do with the Gravitational Field. The issue is how the quadratic congruent symmetries of  $[\mathbb{B}]$  are established, by its group structure. Different choices for the group structure of the Basis Frame will strongly influence the application to any particular physical system.

### **3 Example 1. The Schwarzschild Metric embedded in a Basis Frame. $[\mathbb{B}]$ as a 10 parameter subgroup of an affine group.**

#### **3.1 The metric**

In this example, the isotropic form of the Schwarzschild metric will be incorporated into a Cartan Connection  $[\mathbb{C}]$ . The technique is easily evaluated for diagonal metrics. However,

the symmetry properties of the Cartan Connection are not limited to metrics of the "gravitational" type. Once the Schwarzschild metric is embedded in to the Basis Frame, then the universal methods described above will be applied, and evaluated.

The isotropic Schwarzschild metric is a diagonal metric of the form,

$$(\delta s)^2 = -(1 + m/2r)^4 \{(dx)^2 + (dy)^2 + (dz)^2\} + \frac{(1 - 2m/r)^2}{(1 + 2m/r)^2} (dt)^2 \quad (81)$$

$$= -(\alpha)^2 \{(dx)^2 + (dy)^2 + (dz)^2\} + (\beta)^2 (dt)^2 \quad (82)$$

$$\text{with } r = \sqrt{(x)^2 + (y)^2 + (z)^2}, \quad (83)$$

As Eddington points out, the isotropic form is palatable with the idea that the speed of light is equivalent in any direction. That is not true for the non-isotropic Schwarzschild metric, where transverse and longitudinal null geodesics do not have the same speed.

For the isotropic Schwarzschild example, the metric  $[g_{jk}]$  can be constructed from the triple matrix product:

$$[g_{jk}] = [\tilde{f}] \circ [\eta] \circ [f], \quad (84)$$

$$\text{where } f = \begin{bmatrix} \alpha & 0 & 0 & 0 \\ 0 & \alpha & 0 & 0 \\ 0 & 0 & \alpha & 0 \\ 0 & 0 & 0 & \beta \end{bmatrix}, \quad (85)$$

$$\text{and } \alpha = (1 + m/2r)^2, \quad \beta = \frac{(1 - 2m/r)}{(1 + 2m/r)}, \quad (86)$$

$$\text{and } \eta = \begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}. \quad (87)$$

### 3.2 The Diffeomorphic Jacobian Basis Frame

At first, consider the diffeomorphic map  $\phi^k$  from spherical to Cartesian coordinates:

$$\{y^a\} = \{r, \theta, \varphi, \tau\} \Rightarrow \{x^k\} = \{x, y, z, t\} \quad (88)$$

$$\phi^k : [r \sin(\theta) \cos(\varphi), r \sin(\theta) \sin(\varphi), r \cos(\theta), \tau] \Rightarrow [x, y, z, t] \quad (89)$$

$$\{dy^a\} = \{dr, d\theta, d\varphi, d\tau\}. \quad (90)$$

The Jacobian of the diffeomorphic map  $\phi^k$  can be utilized as an integrable Basis Frame matrix  $[\mathbb{B}]$  which is an element of the 10 parameter F-Affine group (The Affine subgroup with a fixed point):

$$[\mathbb{B}] = \begin{bmatrix} \sin(\theta) \cos(\varphi) & r \cos(\theta) \cos(\varphi) & -r \sin(\theta) \sin(\varphi) & 0 \\ \sin(\theta) \sin(\varphi) & r \cos(\theta) \sin(\varphi) & r \sin(\theta) \cos(\varphi) & 0 \\ \cos(\theta) & -r \sin(\theta) & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}. \quad (91)$$

**Theorem 2** *The effects of a diagonal metric  $[g_{jk}]$  can be absorbed into a re-definition of the Frame matrix:*

$$[\widehat{\mathbb{B}}] = [f] \circ [\mathbb{B}]. \quad (92)$$

The integrable Jacobian Basis Frame matrix given above will be perturbed by multiplication on the left by the diagonal matrix,  $[f]$ . The perturbed Basis Frame becomes

$$[\widehat{\mathbb{B}}] = [f] \circ [\mathbb{B}] \text{ the Schwarzschild Cartan Basis Frame.} \quad (93)$$

$$= \begin{bmatrix} \alpha \sin(\theta) \cos(\varphi) & \alpha r \cos(\theta) \cos(\varphi) & -\alpha r \sin(\theta) \sin(\varphi) & 0 \\ \alpha \sin(\theta) \sin(\varphi) & \alpha r \cos(\theta) \sin(\varphi) & \alpha r \sin(\theta) \cos(\varphi) & 0 \\ \alpha \cos(\theta) & -\alpha r \sin(\theta) & 0 & 0 \\ 0 & 0 & 0 & \beta \end{bmatrix}. \quad (94)$$

Use of the congruent pullback formula based on the perturbed Basis Frame,  $[\widehat{\mathbb{B}}]$ , yields,

$$[g_{mn}] = [\widehat{\mathbb{B}}_{transpose}] \circ \eta \circ [\widehat{\mathbb{B}}], \quad (95)$$

$$= \begin{bmatrix} -\alpha^2 & 0 & 0 & 0 \\ 0 & -\alpha^2 r^2 & 0 & 0 \\ 0 & 0 & -\alpha^2 r^2 \sin^2(\theta) & 0 \\ 0 & 0 & 0 & +\beta^2 \end{bmatrix}, \quad (96)$$

which agrees with formula given above for the isotropic Schwarzschild metric in spherical coordinates. It actually includes a more general idea, for the coefficients,  $\alpha$ , and  $\beta$ , can be dependent upon both  $r$  and  $\tau$ .

### 3.3 The Schwarzschild-Cartan connection.

The Schwarzschild-Cartan (right) Connection  $[\widehat{\mathbb{C}}]$ , as a matrix of 1-forms relative to the Basis Frame  $[\widehat{\mathbb{B}}]$ , becomes

$$[\widehat{\mathbb{C}}] = [\widehat{\mathbb{B}}^{-1}] \circ d[\widehat{\mathbb{B}}], \quad (97)$$

$$[\widehat{\mathbb{C}}] = \begin{bmatrix} -2m dr/r\gamma & -r d\theta & \sin^2(\theta) r d\phi & 0 \\ d\theta/r & \delta dr/\gamma & -\cos(\theta) \sin(\theta) d\phi & 0 \\ d\phi/r & \cot(\theta) d\phi & \cot(\theta) d\theta + \delta dr/\gamma & 0 \\ 0 & 0 & 0 & 4m dr/(\gamma\delta) \end{bmatrix}. \quad (98)$$

$$\gamma = (2r + m), \quad \delta = (2r - m) \quad (99)$$

$$\begin{bmatrix} -2 \frac{d(r) M}{r(2r+M)} & -r d(\theta) & d(\phi) (-1 + \cos(\theta)^2) r & 0 \\ \frac{d(\theta)}{r} & \frac{d(r)(2r-M)}{r(2r+M)} & -d(\phi) \cos(\theta) \sin(\theta) & 0 \\ \frac{d(\phi)}{r} & \frac{d(\phi) \cos(\theta)}{\sin(\theta)} & \frac{d(r)(2r-M)}{r(2r+M)} + \frac{\cos(\theta) d(\theta)}{\sin(\theta)} & 0 \\ 0 & 0 & 0 & 4 \frac{M d(r)}{4r^2 - M^2} \end{bmatrix}$$

#### Perturbed Cartan Connection

Surprisingly, for the perturbed Basis Frame  $[\widehat{\mathbb{B}}]$  representing the field of a massive object, the vector of excitation torsion 2-forms, based on the affine subgroup with a fixed point, are not zero, and can be evaluated as:

$$\left| \widehat{\Sigma}_{torsion\_2-forms} \right\rangle \Rightarrow |\widehat{G}\rangle = [\widehat{\mathbb{C}}]^\wedge |dy^a\rangle \quad (100)$$

$$\text{Torsion 2-forms} \quad : \quad \text{of the Affine subgroup with a fixed point} \quad (101)$$

$$|\widehat{G}\rangle = \left| \begin{array}{l} 0 \\ (2m/r\gamma)(d\theta \wedge dr) \\ (2m/r\gamma)(d\phi \wedge dr) \\ (4m/\gamma\delta)(dr \wedge d\tau) \end{array} \right\rangle \text{"Excitations"} \quad (102)$$

Similarly the perturbed 2-forms of field intensities can be evaluated:

$$|\widehat{F}\rangle = d([\widehat{\mathbb{B}}] \circ |dy^a\rangle) = |dA^k\rangle \quad (103)$$

$$: \text{Intensity 2-forms of the Affine subgroup with a fixed point} \quad (104)$$

$$|\widehat{F}\rangle = \left\langle \begin{array}{l} +(m\gamma/2r^2)\{(\sin(\phi)\cos(\theta)d\theta^{\wedge}dr) - (\sin(\theta)\cos(\phi)d\phi^{\wedge}dr) \\ +(m\gamma/2r^2)\{(\sin(\phi)\cos(\theta)d\theta^{\wedge}dr) + (\sin(\theta)\cos(\phi)d\phi^{\wedge}dr) \\ +(m\gamma/2r^2)(\sin(\theta)d\theta^{\wedge}dr) + \\ (4m/\gamma^2)(dr^{\wedge}d\tau) \end{array} \right\rangle. \text{"Intensities"} \quad (105)$$

The constitutive map relating the field intensities and the field excitations

$$|\widehat{G}\rangle = [\widehat{\mathbb{B}}]^{-1} \circ |\widehat{F}\rangle \quad (106)$$

is determined by the inverse of the perturbed Basis Frame,  $[\widehat{\mathbb{B}}]^{-1}$  :

$$\begin{bmatrix} 4 \frac{r^2 \sin(\theta) \cos(\phi)}{(2r+M)^2} & 4 \frac{r^2 \sin(\theta) \sin(\phi)}{(2r+M)^2} & 4 \frac{r^2 \cos(\theta)}{(2r+M)^2} & 0 \\ 4 \frac{\cos(\theta) r \cos(\phi)}{(2r+M)^2} & 4 \frac{\cos(\theta) r \sin(\phi)}{(2r+M)^2} & -4 \frac{r \sin(\theta)}{(2r+M)^2} & 0 \\ -4 \frac{r \sin(\phi)}{\sin(\theta) (2r+M)^2} & 4 \frac{r \cos(\phi)}{\sin(\theta) (2r+M)^2} & 0 & 0 \\ 0 & 0 & 0 & \frac{2r+M}{2r-M} \end{bmatrix}$$

### Schwarzschild Constitutive Map

The Topological Torsion for the Schwarzschild example vanishes:  $H = \langle A |^{\wedge} |F \rangle \Rightarrow 0$ . The implication is that the system is of Pfaff dimension 2 (and therefore is an equilibrium thermodynamic system). The exterior derivative of the vector of excitations is zero, hence there are no current 3-forms (the existence of currents implies non-equilibrium):

$$\text{Charge Current 3-form } |J\rangle = d|G\rangle = 0. \quad (107)$$

Both Poincare invariants vanish, but the Topological Spin 3-form is NOT zero.

$$\text{Topological Spin 3-form } S = \langle A |^{\wedge} |G \rangle \quad dA = 0. \quad (108)$$

$$= (-m\gamma \sin(\theta)/2r^2)\{\cos(\phi) + 1\}(dr^{\wedge}d\theta^{\wedge}d\phi). \quad (109)$$

The matrices of Connection 1-forms are presented below for each (perturbed) connection,  $[\Gamma]$ ,  $[\mathbb{C}]$ ,  $[\mathbb{T}]$ .

$$\begin{aligned}
 [\Gamma] &= \begin{bmatrix} -2 \frac{d(r)M}{r(2r+M)} & -\frac{(2r-M)r d(\theta)}{2r+M} & \frac{(2r-M)(-1+\cos(\theta)^2)r d(\phi)}{2r+M} & \frac{64 r^4 (2r-M)M d(\tau)}{(2r+M)^7} \\ \frac{(2r-M)d(\theta)}{r(2r+M)} & \frac{d(r)(2r-M)}{r(2r+M)} & -d(\phi)\cos(\theta)\sin(\theta) & 0 \\ \frac{(2r-M)d(\phi)}{r(2r+M)} & -\frac{\cos(\theta)\sin(\theta)d(\phi)}{-1+\cos(\theta)^2} & \frac{d(r)(2r-M)}{r(2r+M)} - \frac{\cos(\theta)\sin(\theta)d(\theta)}{-1+\cos(\theta)^2} & 0 \\ \frac{4M d(\tau)}{(2r-M)(2r+M)} & 0 & 0 & \frac{4M d(\tau)}{(2r-M)(2r+M)} \end{bmatrix} \\
 [C] &= \begin{bmatrix} -2 \frac{d(r)M}{r(2r+M)} & -r d(\theta) & d(\phi)r(-1+\cos(\theta)^2) & 0 \\ \frac{d(\theta)}{r} & \frac{d(r)(2r-M)}{r(2r+M)} & -d(\phi)\cos(\theta)\sin(\theta) & 0 \\ \frac{d(\phi)}{r} & \frac{d(\phi)\cos(\theta)}{\sin(\theta)} & \frac{d(r)(2r-M)}{r(2r+M)} + \frac{\cos(\theta)d(\theta)}{\sin(\theta)} & 0 \\ 0 & 0 & 0 & \frac{4M d(\tau)}{4r^2-M^2} \end{bmatrix} \\
 [T] &= \begin{bmatrix} 0 & -2 \frac{r d(\theta)M}{2r+M} & 2 \frac{d(\phi)r(-1+\cos(\theta)^2)M}{2r+M} & -\frac{64 r^4 (2r-M)M d(\tau)}{(2r+M)^7} \\ 2 \frac{d(\theta)M}{r(2r+M)} & 0 & 0 & 0 \\ 2 \frac{d(\phi)M}{r(2r+M)} & 0 & 0 & 0 \\ -\frac{4M d(\tau)}{4r^2-M^2} & 0 & 0 & 0 \end{bmatrix}
 \end{aligned}$$

### Schwarzschild Perturbed Connections

The matrix of (metric) curvature 2-forms,  $[\Phi_\Gamma]$ , based on the formula

$$[\Phi_\Gamma] = d[\Gamma] + [\Gamma] \wedge [\Gamma], \tag{110}$$

is computed to be:

$$\begin{bmatrix} 0 & -4 \frac{rM(d(r) \wedge d(\theta))}{(2r+M)^2} & 4 \frac{rM(-1+\cos(\theta)^2)(d(r) \wedge d(\phi))}{(2r+M)^2} & -128 \frac{r^3 M(-4rM+M^2+4r^2)(d(r) \wedge d(\tau))}{(2r+M)^8} \\ 4 \frac{M(d(r) \wedge d(\theta))}{r(2r+M)^2} & 0 & -8 \frac{rM(-1+\cos(\theta)^2)(d(\theta) \wedge d(\phi))}{(2r+M)^2} & \frac{64(2r-M)^2 r^3 M(d(\theta) \wedge d(\tau))}{(2r+M)^8} \\ 4 \frac{M(d(r) \wedge d(\phi))}{r(2r+M)^2} & -8 \frac{(d(\theta) \wedge d(\phi))rM}{(2r+M)^2} & 0 & \frac{64(2r-M)^2 r^3 M(d(\phi) \wedge d(\tau))}{(2r+M)^8} \\ -8 \frac{M(d(r) \wedge d(\tau))}{r(2r+M)^2} & -4 \frac{rM(d(\tau) \wedge d(\theta))}{(2r+M)^2} & 4 \frac{rM(-1+\cos(\theta)^2)(d(\tau) \wedge d(\phi))}{(2r+M)^2} & 0 \end{bmatrix}$$

### Curvature 2-forms for the Schwarzschild perturbed Christoffel Connection

By the Strong equivalence Principle,

$$: \{d[\Gamma] + [\Gamma] \wedge [\Gamma]\} + \{[\mathbb{T}] \wedge [\Gamma] + [\Gamma] \wedge [\mathbb{T}]\} + \{d[\mathbb{T}] + [\mathbb{T}] \wedge [\mathbb{T}]\} \quad (111)$$

$$= \{[\Phi_\Gamma]\} + \{[Interaction\ 2 - forms]\} + \{[\Phi_T]\} \Rightarrow 0. \quad (112)$$

which can be checked using Maple algebra.

<http://www22.pair.com/csdc/pdf/mapleep1.pdf>

### 3.4 In Summary

The idea that has been exploited is that the arbitrary Basis Frame (a linear form), without metric, can be perturbed algebraically to produce a new Basis Frame that absorbs the properties of a quadratic metric system. For the Schwarzschild example, another remarkable feature is that the 1-forms  $|\sigma^k\rangle$  constructed according to the formula

$$[\widehat{\mathbb{B}}] \circ |dy^a\rangle \Rightarrow |\sigma^k\rangle = |A^k\rangle, \quad (113)$$

are all integrable (as the Topological Torsion term is Zero). The symbol  $|dy^a\rangle$  stands for the set  $[dr, d\theta, d\varphi, d\tau]$  (transposed into a column vector), and  $[\widehat{\mathbb{B}}]$  is the "perturbed" Schwarzschild metric. The integrability condition means that there exist integrating factors  $\lambda^{(k)}$  for each  $\sigma^k$  such that a new Basis Frame can be constructed from  $[\widehat{\mathbb{B}}]$  algebraically. Relative to this new Basis Frame, the vector of torsion 2-forms is zero,  $|d\sigma^k\rangle = |dA^k\rangle = |F^k\rangle = 0!$  The "Coriolis" acceleration which is related to the 2-form of torsion 2-forms  $|F^k\rangle$  can be eliminated algebraically.! Encroyable! Formidable!

Of course this reduction is impossible if any of the 1-forms,  $\sigma^k$ , is of Pfaff dimension 3 or more. The Basis Frame then admits Topological Torsion, which is irreducible.

## 4 Remarks

This set ideas enumerated in Section 2 startles me. There is only ONE fundamental assumption, and the rest of the 21 equations are derived, following the rules of the Cartan Calculus. It seems to appear that these 21 properties are universal!

*The concepts do not utilize the concept of a Coframe matrix of 1-forms.*

**Frankly, I can use some help and suggestions on how to interpret and apply this "universal" formalism. I am not sure that it is absolutely useful, but it has been my experience and intuition that universal, derived, results (not just**

an Ansatz, or guessing another term to be used in a Lagrangian) generally work out into something useful.

THINGS I WOULD LIKE TO TRY:

**5 Example 2.  $[\mathbb{B}]$  as a 13 parameter intransitive group (Electromagnetism)**

To be worked out using Maple

**6 Example 3.  $[\mathbb{B}]$  as a 13 parameter transitive group (Particles)**

To be worked out using Maple.

**7 Example 4.  $[\mathbb{B}]$  as an Immersion with two parameters (strings).**

To be worked out using Maple.

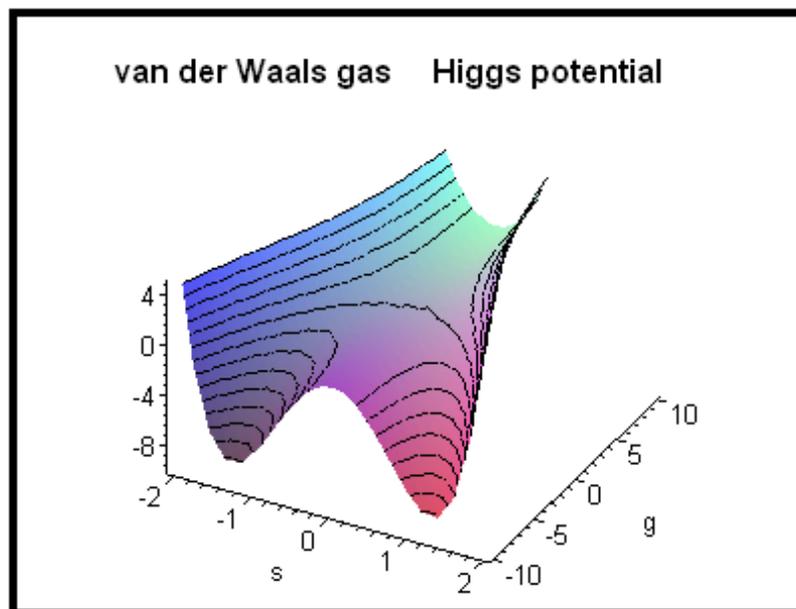
**8 Example 5.  $[\mathbb{B}]$  as a Projection in terms of the Hopf map.**

To be worked out using Maple.

*Note that the metric (matter?) Field parts are related to quadratic forms, while other fields are related to the linear and anti symmetric properties of the connection decomposition (see equation 45).*

## 9 Thermodynamic Phase Functions from [B]

As pointed out in my first volume<sup>2</sup> "Non Equilibrium Thermodynamics and Irreversible Processes", that, given any matrix, useful information (related to Mean and Gaussian Curvatures of the domain described by the matrix) can be obtained from the Cayley-Hamilton Characteristic Polynomial. This polynomial exists for any square matrix. In particular, for 4 x 4 matrices, if the matrix has no null eigenvalues, the domain is then Symplectic and the characteristic polynomial is of 4th degree in the polynomial variable (which represents the eigenvalues). By treating the characteristic polynomial as an expression in the 4D space defined by the similarity invariants of the 4 x 4 matrix, with the polynomial variable (eigenvalue) as a family parameter, the envelope of the reduced quartic polynomial yields a thermodynamic phase function that establishes an extraordinary correspondence between the binodal, spinodal lines of a van der Waals gas, and its critical point. These curves act as bifurcation limit sets on the quartic surface; the critical point is at  $s = g = k = 0$ . I call this envelope, the Higgs Phase function. The conjecture is that Higgs features (related to mass and inertial) have a basis in topological thermodynamics, for at the critical point it is known from chemical thermodynamics (and Lev Landau) that there are large fluctuations in density, as the material attempts to condense, and these fluctuations are correlated with a  $1/r^2$  force law.



<sup>2</sup>See <http://www.lulu.com/kieln>

It is remarkable to me that all of this starts from the sole assumption of a Basis Frame [B]. The Higgs idea and the Yang Mills theory (and weak force) seem to be related to an irreducible Pfaff topological dimension 4 (Evolutionary irreversible processes are possible and do not preserve parity).

I conjecture that if the Phase function has 1 null eigen value, then the space is related to a non equilibrium configuration of Pfaff dimension 3, for which parity is always preserved (the strong Force). The electromagnetic domain only requires Pfaff dimension 2, and the gravity domain is embedded in Pfaff topological dimension 1. This follows from an old I argument (based on differential geometry) that I presented in 1975.

Kiehn, R. M. (1975), "Submersive equivalence classes for metric fields",  
Lett. al Nuovo Cimento 14, p. 308.

The idea is that spaces of Pfaff Topological dimension 2 or less (hence thermodynamically in equilibrium) are topologically *connected*, and interactions can range over the entire domain (long range forces - gravity and electromagnetism). However, spaces of Pfaff Dimension 3 or more are topologically *disconnected* domains of multiple components, and are not in thermodynamic equilibrium. Hence interactions and forces are short range (strong and weak forces, that may or may not preserve parity)

## 9.1 Comparisons and Differences.

When F. Hehl suggested I look at recent work concerning Metric Affine Gravity theories, and Coframe theories, a quick scan of the literature made me feel that much of what is suggested in Section 2 above is more or less covered within these concepts of MAG. However, a closer reading, indicates that emphasis is indeed placed on a Basis Frame, but the coframe is defined as a matrix set of set of 1-forms,  $\theta^\beta$ , that are "orthonormal" with respect to vectors that make up the frame field as a matrix of functions,  $e_\alpha$ .

The method I employ starts with a frame field which I denote as [B]. The Basis Frame matrix is a matrix of C2 functions. An orthonormal set of 1-forms,  $\theta^\beta$ , is not utilized, nor needed.

## 10 References

### References

- [1] Itin, Y. (2005) "Vacuum Electromagnetism from a geometrical structure", arXiv:gr-qc/0510075

- [2] Marsden, J.E. and Riatu, T. S. (1994) "Introduction to Mechanics and Symmetry", Springer-Verlag, p.122
- [3] Mason, L. J., and Woodhouse, N. M. J. Woodhouse, (1996), "Integrability, Self Duality, and Twistor Theory", Clarendon Press, Oxford.
- [4] Kiehn, R.M. and Pierce, J. F., (1969), An Intrinsic Transport Theorem, Phys. Fluids, 12, p.1971. (<http://www22.pair.com/csdc/pdf/inttrans.pdf>)
- [5] Kiehn, R.M. (1974), Extensions of Hamilton's Principle to Include Dissipative Systems, J. Math Phys. 15, p. 9
- [6] Kiehn, R.M. (1975), Intrinsic hydrodynamics with applications to space-time fluids, Int. J. of Eng. Sci. 13, 941. (<http://www22.pair.com/csdc/pdf/inthydro.pdf>) arXiv/math-ph/0101032
- [7] Kiehn, R. M., (1975), Geometrodynamic and Spin", Lett. al Nuovo Cimento 12, p. 300
- [8] Kiehn, R. M. (1975), Submersive equivalence classes for metric fields, Lett. al Nuovo Cimento 14, p. 308
- [9] Kiehn, R.M. (1975), Conformal Invariance and Hamilton Jacobi theory for dissipative systems, J. Math Phys. 16, 1032
- [10] Kiehn, R.M. (1976), NASA AMES NCA-2-OR-295-502  
(<http://www22.pair.com/csdc/pdf/rmkames.pdf>)
- [11] R. M. Kiehn, Bull Am. Phys Soc., **12**, 198 (1967). For source free systems in four dimensions, see: R. Debever, Coll. de Geom. Diff., Louvain, 217, (1951), and C.W.Misner and J. A. Wheeler, Ann. Phys. (N. Y.) **2**, 525 (1957)
- [12] Kiehn, R.M. (1976), Retrodictive Determinism, Int. J. of Eng. Sci. 14, 749. ([../retrodic.pdf](http://www22.pair.com/csdc/pdf/retrodic.pdf))
- [13] Kiehn, R.M. (1977), Periods on manifolds, quantization and gauge, J. Math Phy. 18, 614. ([../periods.pdf](http://www22.pair.com/csdc/pdf/periods.pdf))
- [14] Sanders, V. E. and Kiehn, R.M. (1977), Dual Polarized Ring Lasers, IEEE J Quant Elec QE-13, p. 739

- [15] The explicit variation of the helicity density was reported by R.M.Kiehn as a reason for the transition from laminar to non-laminar flow in the NASA-AMES research document NCA-2-0R-295-502 (1976). Also see R. M. Kiehn, R. M. (1977) *J, Math Phys.* 18 p. 614, and R. M. Kiehn, R. M. (1975) *Intrinsic hydrodynamics with applications to space time fluids., Int. J. Engng Sci.* 13 p. 941-949.
- [16] The importance of the N-1 form  $A^H$  (now written as  $A^G$ ) was first anticipated in: R. M. Kiehn and J. F. Pierce, *An Intrinsic Transport Theorem Phys. Fluids* 12, 1971. The concept was further employed in R. M. Kiehn, *Periods on manifolds, quantization and gauge, J. of Math Phys* 18 no. 4 p. 614 (1977), and with applications described in R. M. Kiehn, *Are there three kinds of superconductivity, Int. J. Mod. Phys B* 5 1779. (1991)
- [17] Kiehn, R. M., (1987) The Falaco Effect as a topological defect was first noticed by the present author in the swimming pool of an old MIT friend, during a visit in Rio de Janeiro, at the time of Halley's comet, March 1986. The concept was presented at the Austin Meeting of Dynamic Days in Austin, January 1987, and caused some interest among the resident topologists. The easily reproduced experiment added to the credence of topological defects in fluids. It is now perceived that this topological phenomena is universal, and will appear at all levels from the microscopic to the galactic. LANL arXiv/gr-qc/0101098 (../falaco97.pdf and../topturb.pdf)
- [18] Sterling. et.al. (1987), Why are these disks dark? *Phys Fluids* 30 p.11
- [19] Kiehn, R. M. (1987), Falaco Solitons, ../falaco01.pdf
- [20] Unpublished talk presented at the January 1987 Dyne Days conference in Austin, Texas. The long lifetime and the Snell refractlon torsion surface of negative curvature was first appreciated by this author on March 27, 1986 while in the swimming pool of an old MIT friend in Rio de Janeiro.
- [21] Kiehn, R. M., (1989), Irreversible Topological Evolution in Fluid Mechanics in "Some Unanswered Questions in Fluid Mechanics" ASME- Vol. 89-WA/FE-5, Trefethen, L. M. and Panton, R. L. Eds.
- [22] Kiehn, R.M, (1987) "The Falaco Effect, A Topological Soliton" Talk presented at the Dynamics Days conference, Austin, TX Jan. 1987. The Falaco Effect as a topological defect was first noticed by the present author in the swimming pool of an old MIT friend, during a visit in Rio de Janeiro, at the time of Halley's comet, March 1986. The concept was presented at the Austin Meeting of Dynamic Days in Austin, January

1987, and caused some interest among the resident topologists. The easily reproduced experiment added to the credence of topological defects in fluids. It is now perceived that this topological phenomena is universal, and will appear at all levels from the microscopic to the galactic.

- [23] Kiehn, R. M., (1988 ) "Torsion in Crystalline Fluids" The Energy Laboratory Newsletter, 18 Univ. of Houston, p.6
- [24] First presented at the January 1990 Dynamics Days, conference at Austin, Texas. Submitted for publication
- [25] Kiehn, R.M. (1990), Topological Torsion, Pfaff Dimension and Coherent Structures, in: "Topological Fluid Mechanics", H. K. Moffatt and T. S. Tsinober eds, Cambridge University Press p. 449-458. (../camb89.pdf)
- [26] Kiehn, R. M. (1991), Compact Dissipative Flow Structures with Topological Coherence Embedded in Eulerian Environments, in: " non linear Dynamics of Structures", edited by R.Z. Sagdeev, U. Frisch, F. Hussain, S. S. Moiseev and N. S. Erokhin, p.139-164, World Scientific Press, Singapore
- [27] Kiehn, R.M. (1991) "Continuous Topological Evolution", LANL arXiv/math-ph/0101032 (../contevol3.pdf)
- [28] Kiehn, R. M. (1991), Are there three kinds of superconductivity, Int. Journ. of Modern Physics, vol5 #10 p. 1779
- [29] Kiehn, R. M., Kiehn, G. P., and Roberds, (1991), Parity and time-reversal symmetry breaking, singular solutions and Fresnel surfaces, Phys. Rev A 43, pp. 5165-5671. (../timerev.pdf)
- [30] Kiehn, R.M. (1991) Some Closed Form Solutions to the Navier Stokes Equations, LANL arXiv/physics/0102002 (../nvsol.pdf)
- [31] Kiehn, R.M. (1991), "Dissipation, Irreversibility and Symplectic Lagrangian systems on Thermodynamic space of dimension  $2n+2$ ", ../irrev1.pdf
- [32] Kiehn, R. M., "Topological Parity and the Turbulent State of a Navier-Stokes Fluid", submitted for publication in the Jap Journ. of Fluid Res.
- [33] Kiehn, R. M. (1992), Topological Defects, Coherent Structures and Turbulence in Terms of Cartan's Theory of Differential Topology, in "Developments in Theoretical and Applied Mathematics, Proceedings of the SECTAM XVI conference", B. N. Antar, R.

Engels, A.A. Prinaris and T. H. Moulden, Editors, The University of Tennessee Space Institute, Tullahoma, TN 37388 USA.

- [34] Kiehn, R. M. (1993), Instability Patterns, Wakes and Topological Limit Sets, in "Eddy Structure Identification in Free Turbulent Shear Flows", J.P.Bonnet and M.N. Glauser, (eds), Kluwer Academic Publishers p. 363.
- [35] Kiehn, R. M. (1993) Talk presented at the EUROMECH 308 meeting at Cortona, Italy. Unpublished. (../italy.pdf)
- [36] Kiehn, R. M. (1995), Hydrodynamic Wakes and Minimal Surfaces with Fractal Boundaries, in "Mixing in Geophysical Flows", J. M. Redondo and O. Metais, Editors CIMNE, Barcelona ERCOFTAC (meeting in 1992) p. 52-63.
- [37] Kiehn, R. M. (1997) "When does a dynamical system represent an Irreversible Process" SIAM Snowbird May 1997 poster.
- [38] Kiehn, R. M., (1999), Coherent Structures in Fluids are Topological Torsion Defects, in J, "IUTAM Symposium on Simulation and Identification of Organized Structures in Flows", N. Sørensen, et al., eds., Kluwer Academic Publishers, Dordrecht,. See (../copen5.pdf). Presented at the IUTAM-SIMFLO Conference at DTU, Denmark, May 25-29, (1997).
- [39] Kiehn, R.M. (1999), Topological evolution of classical electromagnetic fields and the photon, in, "Photon and Poincaré Group", V. Dvoeglazov (ed.), Nova Science Publishers, Inc., Commack, New York, p.246-262. ISBN 1-56072-718-7. Also see (../photon5.pdf)
- [40] Kiehn, R. M. (2000), 2D turbulence is a Myth, (invited speaker EGS XXIV General Assembly IUTAM, the Hague, 1999 ../hague6.pdf)
- [41] Kiehn, R.M., (2001) Topological-Torsion and Topological-Spin as coherent structures in plasmas", arXiv physics /0102001. ../plasmas.pdf, and ../holder3d.pdf)
- [42] Kiehn, R. M. (2002), Curvature and torsion of implicit hypersurfaces and the origin of charge, Annales de la Fondation Louis de Broglie, vol 27, p.411. LANL arXiv /gr-qc /0101109 (../contevol3.pdf)
- [43] Kiehn, R.M. (2002), The Photon Spin and other Topological Features of Classical Electromagnetism, in "Gravitation and Cosmology, From the Hubble Radius to the Planck

- Scale", Amoroso, R., et al., eds., Kluwer, Dordrecht, Netherlands, p.197-206. Vigier 3 conference in 2000. (../vig2000.pdf)
- [44] Kiehn, R. M. (2003), Thermodynamic Irreversibility and the Arrow of Time, in "The Nature of Time: Geometry, Physics and Perception", R. Bucher et al. (eds.), Kluwer, Dordrecht, Netherlands, p.243-250. (../arwfinal.pdf)
- [45] Kiehn, R. M. (2004), A topological perspective of Electromagnetism, to be published in "Classical electrodynamics: new horizons", Editors Andrew E. Chubykalo and Roman Smirnov-Rueda, Rinton Press, Inc. USA. Also see../rmktop.pdf
- [46] Kiehn, R. M. (2004?) "A Topological Perspective of Cosmology." to be published. Also see../cosmos.pdf
- [47] Kiehn, R. M. (2004?) "A Topological Perspective of Plasmas" to be published. Also see../plasmanew.pdf
- [48] see *Holder Norms (3D)*,../holder3d.pdf *Holder Norms (4D)*,../holder4d.pdf *Implicit surfaces*,../implinor.pdf in Kiehn, R. M. (2004) Exterior Differential Forms and Differential Topology, "Non Equilibrium Systems and Irreversible Processes Vol 5"
- [49] see *Dissipation, Irreversibility and Symplectic Lagrangian Systems on Thermodynamic Space of Dimension  $2n+2$* ,../irrev1.pdf in Kiehn, R. M. (2004) Exterior Differential Forms and Differential Topology, "Non Equilibrium Systems and Irreversible Processes Vol 5"
- [50] Schouten, J. A. and Van der Kulk, W. (1949) "Pfaff's Problem and its Generalizations", Oxford Clarendon Press.
- [51] Shipov, G., (1998), "A Theory of Physical Vacuum", Moscow ST-Center, Russia ISBN 5 7273-0011-8
- [52] H. W. Turnbull, H. W. (1960) "The Theory of determinants, matrices and invariants" Dover, New York (1934)
- [53] H. W. Turnbull, H. W and Aitken, A.C. (1961) "An Introduction to the theory of Canonical Matrices" Dover, New York (1932)
- [54] Kiehn, R. M. (2004) Non Equilibrium Thermodynamics, "Non Equilibrium Systems and Irreversible Processes Vol 1" see <http://www.cartan.pair.com>

- [55] Kiehn, R. M. (2004) Cosmology, Falaco Solitons and the Arrow of Time, "Non Equilibrium Systems and Irreversible Processes Vol 2" see <http://www.cartan.pair.com>
- [56] Kiehn, R. M. (2004) Wakes, Coherent Structures and Turbulence, "Non Equilibrium Systems and Irreversible Processes Vol 3" see <http://www.cartan.pair.com>
- [57] Kiehn, R. M. (2004) Plasmas and Non equilibrium Electrodynamics, "Non Equilibrium Systems and Irreversible Processes Vol 4" see <http://www.cartan.pair.com>
- [58] Kiehn, R. M. (2004) Exterior Differential Forms and Differential Topology, "Non Equilibrium Systems and Irreversible Processes Vol 5" see <http://www.cartan.pair.com>
- [59] Kiehn, R. M. (2004) Maple Magic and Differential Topology "Non Equilibrium Systems and Irreversible Processes Vol 6" see <http://www.cartan.pair.com>