LOCAL NATURE OF INERTIAL PROPERTIES OF MASS IN DESCARTESIAN MECHANICS

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1 Inertial Mass in Descartesian Mechanics. Four-Dimensional Gyroscope

The inertial rest mass of an object in Descartesian mechanics is defined as

$$m_0 = \int \rho(-g)^{1/2} dV, \tag{1}$$

where

$$g = \det g_{jm}, \qquad dV = dx^1 dx^2 dx^3,$$

and the density ρ is defined according to $\rho = 2g_{jm}(\nabla_{[i}T^{i}{}_{|j|m]} + T^{i}{}_{s[i}T^{s}{}_{|j|m]})/\nu c^2$. The final expression of the inertial rest mass in Descartesian mechanics looks

$$m_0(x_0) = \frac{2}{\nu c^2}\int (-g)^{1/2}\left\{g^{jm}\left(\nabla_{[i}T^{i}_{|j|m]} + T^{i}{}_{s[i}T^{s}_{|j|m]}\right)\right\} dV. \tag{2}$$

The relationships show that the inertial rest mass in Descartesian mechanics represents *the measure of the inertial field* $T^{i}{}_{jk}$. Since the inertial field $T^{i}{}_{jk}$ originated by the rotation of the matter (according to E. Cartan) , then the inertial properties of the rest mass depend on the conditions of the rotation of the matter, forming the discussed system. For example, by changing the angular velocity of the separate mass parts of the system $m_0(t)$ according to a certain law, then we can create a "jet-like motion without rejecting the mass"according to the motion equations

$$m_0(t)\frac{d}{dt}(v_\alpha) = -v_\alpha \frac{d}{dt} m_0(t), \quad \alpha = 1, 2, 3. \tag{3}$$

The mechanical device, where the center of mass moves according to the equations (3), has been called a four-dimensional gyroscope (4-D -gyroscope) (see fig. 1)

All the elements of the conventional 3-dimensional gyroscope rotate in spatial angle ϕ in the planes, perpendicular to the axis of rotation. A 4-D gyroscope consists of three connected masses (see 1), two of which (masses m) rotate synchronously in different directions in spatial angle $\phi(t)$ around axis O_1, set on the central mass M. The central mass M itself oscillates along axis of symmetry x with the acceleration

$$W_x = \frac{dv_x(t)}{dt} = c(th\dot{\theta}_x) = c\frac{d\,[th\ \theta_x(t)]}{dt},$$

where θ – pseudo-Euclidean angle. That is why in Descartesian mechanics in terms of Lorentz local group the rotation of 4-D gyroscope is described by two matrixes. Particularly the spatial rotation of the small masses m defined by matrix

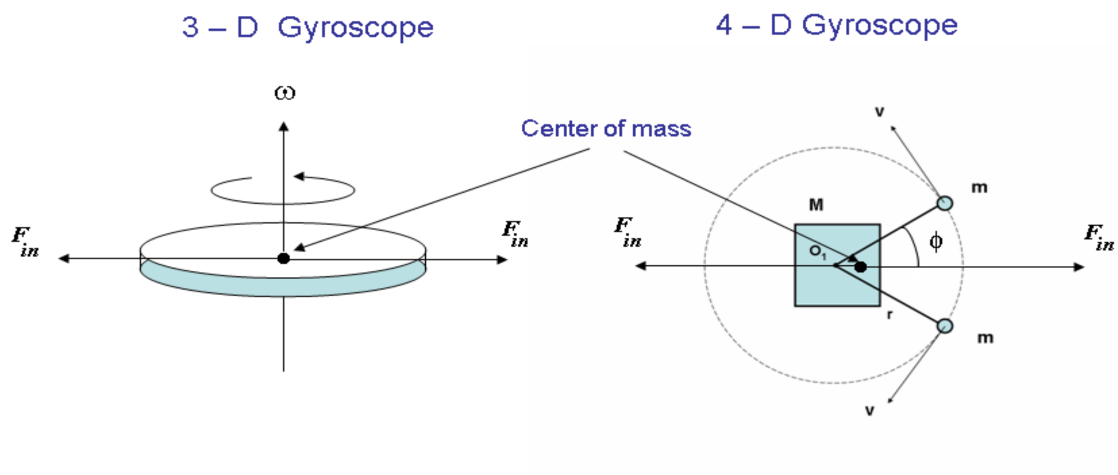

Рис. 1: The local inertial reference frame of the second kind (in which forces of inertia compensate each other) is connected with the center of masses of 3-D and 4-D free gyroscopes

$$R = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos\phi(t) & \sin\phi(t) & 0 \\ 0 & -\sin\phi(t) & \cos\phi(t) & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix},$$

and accelerated motion along axis x is described by the matrix

$$L = \begin{pmatrix} 1 & -th\ \theta(t)_x & 0 & 0 \\ -th\ \theta(t)_x & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}.$$

So, all motions of 4-D gyroscope are rotation (Rene Descartes idea is all motions in the Reality are rotations). Thus it becomes clear, why such a simple mechanical device had been called 4-D gyroscope.

Lagrangian function T of 4-D gyroscope can be presented as

$$T = \frac{M+2m}{2}\left(v_c^2 + k^2(1-k^2\sin^2\phi)w^2\right) = \frac{M+2m}{2}\left(v_c^2 + g'w^2\right), = \frac{M+2m}{2}\dot{s}^2 \qquad (4)$$

where

$$w = r\omega, \quad k^2 = 2m/(M+2m), \quad v_c = v - k^2 w\sin\phi, \quad g' = k^2(1-k^2\sin^2\phi) = k^2 g.$$

Here v_c - velocity of the center of masses, v - the velocity of the central mass M, $\omega = \dot{\phi}$ - angular velocity of the rotation of small masses, r – distance from O_1 to small masses m. Let us consider, that the motion of the center of masses free from external forces of a 4-D gyroscope occurs according to the motion equations of Descartesian mechanics

$$\frac{d^2x^i}{ds^2} + \Delta^i{}_{jk}\frac{dx^j}{ds}\frac{dx^k}{ds} = 0, \tag{5}$$
$$i, j, k... = 1, 2$$

where

$$\Delta^i{}_{jk} = \Gamma^i{}_{jk} + T^i{}_{jk} = e^i{}_a e^a{}_{j,k}.$$

$$ds^2 = g_{ij}dx^i dx^j = \frac{2T}{M+2m}dt^2, \qquad i, j = 1, 2,$$
$$g_{ij} = \begin{pmatrix} 1 & 0 \\ 0 & g' \end{pmatrix} = \Lambda_{ab}e^a{}_i e^b{}_j, \tag{6}$$
$$\Lambda_{ab} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}.$$

The orthogonal diad $e^a{}_i$ for the given metric tensor connected with the variables

$$v_c(t) = \cos\eta(t)\dot{s}, \quad \sqrt{g'}w(t) = \sin\eta(t)\dot{s}, \tag{7}$$

and can be viewed as

$$e^a{}_i(\eta(t)) = \begin{pmatrix} \cos & \sqrt{g'}\sin\eta \\ -\sin\eta & \sqrt{g'}\cos\eta \end{pmatrix},$$
$$e^i{}_a(\eta(t)) = \begin{pmatrix} \cos\eta & -\sin\eta \\ \frac{1}{\sqrt{g'}}\sin\eta & \frac{1}{\sqrt{g'}}\cos\eta \end{pmatrix}.$$

After corresponding calculation, the motion equations will become

$$\frac{dv_c}{dt} = \frac{2m}{M+2m}\Phi\omega, \tag{8}$$

$$\frac{d\omega}{dt} - k^2\omega^2\frac{\sin\phi\cos\phi}{1-k^2\sin^2\phi} = -\frac{1}{rg}\Phi v_c, \tag{9}$$

where

$$\Phi(t) = -\frac{\sqrt{g'}}{k^2}\frac{d\eta}{dt} \tag{10}$$

- function, created by Ricci torsion. If this function goes to zero, then the equations (9) and (10) coincide with the motion equations of 4-D gyroscope, which follow from Newtonian mechanics.

2 Space-Time Precession of a Free Four-Dimensional Gyroscope

The equations (9) and (10) could be presented as

$$\dot{v}_c^* = k^2\Phi^* w,$$

$$\dot{w} = -\Phi^* v_c^*.$$

where

$$v_c^* = v_c - v_0, \qquad \Phi^* = \frac{\Phi}{\sqrt{g'}},$$

and $v_0 = const$ – the initial velocity of the center of mass.

Suppose, that

$$\Phi^* = \kappa_0 = const,$$

then we will obtain the following special solutions for the motion equations

$$v_c(t) = v_0 \sin(k\kappa_0 t) + v_0 = v_0 \left(1 + \sin(k\kappa_0 t)\right), \tag{11}$$

$$\omega(t) = \frac{v_0}{\sqrt{g'} r k} \cos(k\kappa_0 t)) + \frac{r\omega_0 \sqrt{g'(\phi_0)} - v_0/k}{r\sqrt{g'}},$$

which shows , that if the initial velocity of the center of mass different from zero there is the space-time precession of the 4-D gyroscope, *which manifests in changing the velocity of the center of mass of the 4-D gyroscope, when it is free from the action of external forces*!!!

Conclusion:

Controlling space-time precession of 4-D gyroscope, we can create a "jet-like motion without rejecting the mass"according to the motion equations

$$\mathbf{m_0(t)} \frac{\mathbf{d}}{\mathbf{dt}} \left(\mathbf{v}_\alpha\right) = -\mathbf{v}_\alpha \frac{\mathbf{d}}{\mathbf{dt}} \mathbf{m_0(t)}, \quad \alpha = 1, 2, 3.$$