

Descartesian Mechanics: The Fourth Generalization of Newton's Mechanics

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Abstract *The fourth generalization of Newton's Mechanics is considered. The oriented material point became a principle object for the study, while in Newton mechanics it was just a point. The space-time in new mechanics is represented by 10 dimensional fibre bundle, where 4 translational coordinates form base and 6 anholonomic angular - a fibre. The principle consequence of the new mechanics is the connection between the general relativity theory and quantum mechanics. In non relativistic approach it is possible to establish the theoretical foundation of "jet like motion without rejection of mass". This conclusion was verified by experimental results with 4-D Gyroscope.*

Introduction

For 317 years we have been applying Newton's mechanics to explain non-relativistic mechanical experiments on the "bench table". Although Newton's mechanics has been generalized three times: by the special relativity theory, general relativity theory, and quantum mechanics, there remains a possibility for its further generalization.

1 Frenet's Oriented Point

Newton's mechanics as well as all its generalizations, mentioned above, have been based upon the concept of the material point, substituting all the material bodies in this theory. The exception is Quantum Mechanics, where the material particles demonstrate both their corpuscular and wave properties. In the three-dimensional reference frame a material point has three degrees of freedom (according to the number of coordinates). In 1847 F. Frenet introduced for the first time the concept of an "oriented point", connected with three orthogonal unit vectors, orienting it. In the three-dimensional coordinate space x_α , ($\alpha = 1, 2, 3$) the oriented point has got six degrees of freedom - three translational and three rotational [1].

In arbitrary coordinate system and in modern notations, the Frenet's motion equations for the three-dimensional oriented point could be written as [2]

$$\frac{De^A_\alpha}{ds} = T^A_{B\gamma} e^B_\alpha \frac{dx^\gamma}{ds} \quad \text{or} \quad \frac{de^A_\alpha}{ds} = \Delta^A_{B\gamma} e^B_\alpha \frac{dx^\gamma}{ds}, \quad (1)$$

$$\alpha, \beta, \gamma \dots = 1, 2, 3, \quad A, B, C \dots = 1, 2, 3,$$

where $\alpha, \beta, \gamma \dots$ - vectors' induces and induces $A, B, C \dots$ - denote vectors of the Frenet's triad,

$$ds^2 = g_{\alpha\beta} dx^\alpha dx^\beta = \eta_{AB} e^A_\alpha e^B_\beta dx^\alpha dx^\beta, \quad \eta_{AB} = \eta^{AB} = \text{diag}(1 \ 1 \ 1) \quad (2)$$

- the square of the element of the curve's length, where the oriented point moves along, D
- is absolute differential with respect to the Christoffel symbols

$$\Gamma^\alpha_{\beta\gamma} = \frac{1}{2} g^{\alpha\eta} (g_{\beta\eta,\gamma} + g_{\gamma\eta,\beta} - g_{\beta\gamma,\eta}). \quad (3)$$

Magnitudes

$$T^A_{B\gamma} = \nabla_\gamma e^A_\alpha e_B^\alpha = e_{\alpha,\gamma}^A e_B^\alpha - \Gamma_{\alpha\gamma}^\beta e_\beta^A e_B^\alpha = \Delta^A_{B\gamma} - \Gamma^A_{B\gamma} \quad (4)$$

had been introduced by F. Ricci [3] and named later as the Ricci rotation coefficients and the geometric object

$$\Delta^A_{B\gamma} = \Gamma^A_{B\gamma} + T^A_{B\gamma} = e_{\alpha,\gamma}^A e_B^\alpha = \frac{\partial e_\alpha^A}{\partial x^\gamma} e_B^\alpha \quad (5)$$

- connection of absolute parallelism [4].

The Ricci rotation coefficients $T^A_{B\gamma}$ describes the changes of the orientation of basic vectors e_B^α and define the rotational metric [2]

$$d\nu^2 = e^\beta_A D e^A_\alpha e^\alpha_A D e^A_\beta = T^A_{B\alpha} T^B_{A\beta} dx^\alpha dx^\beta, \quad (6)$$

If we select the right triad e_α^A so that unit vectors $e_\alpha^{(1)} = dx_\alpha/ds$, $e_\alpha^{(2)}$ and $e_\alpha^{(3)}$, will be correspondingly a tangent, normal and binormal to the curve, then the equations (1), written in the Descartesian reference frame, will lead to the Frenet's equations

$$\frac{d\mathbf{e}^{(1)}}{ds} = \kappa(s)\mathbf{e}^{(2)}, \quad (7)$$

$$\frac{d\mathbf{e}^{(2)}}{ds} = -\kappa(s)\mathbf{e}^{(1)} + \chi(s)\mathbf{e}^{(3)}, \quad (8)$$

$$\frac{d\mathbf{e}^{(3)}}{ds} = -\chi(s)\mathbf{e}^{(2)}, \quad (9)$$

where $\kappa(s)$ - curvature and $\chi(s)$ - torsion of curve are connected with Ricci rotational coefficients $T^A_{B\gamma}$ in the following way

$$\kappa(s) = T^{(1)}_{(2)\gamma} \frac{dx^\gamma}{ds}, \quad \chi(s) = T^{(2)}_{(3)\gamma} \frac{dx^\gamma}{ds}. \quad (10)$$

From the equations (7)-(9) we will get the translational motion equations of the oriented point (motion equations of the origin of triad)

$$\frac{d^2\mathbf{x}}{ds^2} = \kappa(s)\mathbf{e}^{(2)}, \quad (11)$$

$$\frac{d^3\mathbf{x}}{ds^3} = \frac{d\kappa(s)}{ds}\mathbf{e}^{(2)} - \kappa^2(s)\mathbf{e}^{(1)} + \kappa(s)\chi(s)\mathbf{e}^{(3)}. \quad (12)$$

If we multiply the equations (11) by total mass m of the oriented point, then we shall get the equations similar to the motion equations of Newton's mechanics

$$m \frac{d^2 \mathbf{x}}{ds^2} = \mathbf{F} , \quad (13)$$

where

$$\mathbf{F} = m \kappa(s) \mathbf{e}^{(2)} \quad (14)$$

- force, causing translational acceleration. From the above we can see, that the mechanics of the oriented point can generalize Newton's mechanics as well, allowing us:

- a) To view the dynamics of the physical objects as rotation (Descartes' idea):
- b) To consider the "inner" degrees of freedom, connected with its own rotation of the oriented point, that are not addressed in Newton's mechanics.

2 Clifford's Program on Geometrization of Physics

The curvature and torsion in the Frenet's equations uniformly define the arbitrary curve in three-dimensional reference frames. If we compare the Frenet's curve with a certain physical path, then it will allow us to describe the motion of the material point, which may change its orientation in space. We will call such an object as the "oriented material point". Let the curve $\kappa(s)$ in the Frenet's equations be equal to zero, then it follows from (11)-(14) the force acting upon the oriented material point is absent and it moves straight along the line. Meanwhile its orientation in space changes according to the equations

$$\frac{d\mathbf{e}^{(1)}}{ds} = 0 , \quad \frac{d\mathbf{e}^{(2)}}{ds} = \chi(s) \mathbf{e}^{(3)} , \quad \frac{d\mathbf{e}^{(3)}}{ds} = -\chi(s) \mathbf{e}^{(2)} . \quad (15)$$

Thus, these equations describe their own rotations of the oriented point, affected by the rotational field $\chi(s)$ - torsion field [2], while the action is forceless. The equations (13) are interesting, because they allow to find the geometrical description for physical interactions, which are based upon the Newton's equations. In order to do so it will be sufficient to select the curvature $\kappa(s)$ related to (14). Perhaps, the similar ideas led Clifford saying in 1870 that "there is nothing happening in the world, except changes of the space curvature" [5]. However, being consistent, we could refine it by saying: "there is nothing happening in the world, except changes of the space curvature and torsion of space". To prove it with the help of the Frenet's equations - is impossible. These equations describe just an arbitrary curve in the three-dimensional space. Moreover, it would be a better idea to call $\kappa(s)$ and $\chi(s)$ as the first and second torsion of a curve, since they are defined through the Ricci rotation coefficients $T^A_{B\gamma}$ according to the relations (10). It is understood that the geometrization of physics requires such a geometry, which has got the Riemann curvature and torsion, created by the Ricci rotation coefficients.

3 Ricci's Curvature on Manifold of Oriented Points

We know that Riemann applied point manifold to define the curvature tensor R^i_{jkm} of non-Euclidean space. Ricci in his work [3] finds for the first time the curvature tensor

for the manifold of the oriented points. To be more exact and guided by the physical applications, let us write the principal formulas from Ricci's work [3] for the manifold of the oriented points with 4-dimensions, using modern notations. The generalization for a larger number of dimensions is not difficult. Following Ricci, let us consider four - dimensional differentiated manifold with coordinates x^i ($i = 0, 1, 2, 3$). In each point of this manifold there are - vector e^a_i ($i = 0, 1, 2, 3$) and co vector e^j_b ($b = 0, 1, 2, 3$) with the normalization conditions

$$e^a_i e^j_a = \delta^j_i, \quad e^a_i e^i_b = \delta^a_b. \quad (16)$$

With such a task the four coordinates x^i describe the origin O of four-dimensional oriented point (tetrad), and six independent (due to the conditions of (16)) components of tetrad e^a_i describe its space orientation, playing the role of angular variables.

Tetrad e^a_i defines the metric tensor of space

$$g_{ik} = \eta_{ab} e^a_i e^b_k, \quad \eta_{ab} = \eta^{ab} = \text{diag}(1 \ -1 \ -1 \ -1) \quad (17)$$

and Riemannian (translational) metric

$$ds^2 = g_{ik} dx^i dx^k. \quad (18)$$

Moreover the covariant derivatives of e^a_i along coordinates x_i define the Ricci rotation coefficients [3]

$$T^i_{jk} = e^i_a \nabla_k e^a_j = -\Omega_{jk}^{\cdot i} + g^{im} (g_{js} \Omega_{mk}^{\cdot s} + g_{ks} \Omega_{mj}^{\cdot s}), \quad (19)$$

where ∇_k is the covariant derivative with respect to the Christoffel's symbols

$$\Gamma^i_{jk} = \frac{1}{2} g^{im} (g_{jm,k} + g_{km,j} - g_{jk,m}), \quad (20)$$

and the quantity [3]

$$\Omega_{jk}^{\cdot i} = e^i_a e^a_{[k,j]} = -\frac{1}{2} e^i_a (e^a_{j,k} - e^a_{k,j}) = -T^i_{[jk]} \quad (21)$$

has been called by J. Schouten as an object of anholonomy [6]. This name had been justified by the fact that six angular variables, orienting the triad, are anholonomic. Naturally, when the object of anholonomy (21) goes to zero, there will be no change for the orientation of a point. If the orientation of tetrad vectors changes, then we get the rotational metric [2]

$$d\tau^2 = T^i_{jk} T^j_{in} dx^k dx^n, \quad (22)$$

which describes the infinitesimal turn. Further Ricci demonstrates [3] that there are two curvature tensors for the manifolds of the oriented points:

a) Tensor of Riemannian curvature, defined through Christoffel's symbols by conventional way

$$R^i_{jkm} = 2\Gamma^i_{j[m,k]} + 2\Gamma^i_{s[k} \Gamma^s_{j|m]}; \quad (23)$$

b) Tensor of Ricci curvature, defined through the Ricci rotation coefficients as

$$P^i_{jkm} = 2\nabla_{[k} T^i_{j|m]} + 2T^i_{c[k} T^c_{j|m]}. \quad (24)$$

Because the sum $\Gamma_{jk}^i + T_{jk}^i$ forms the connection geometry of absolute parallelism [7]

$$\Delta_{jk}^i = \Gamma_{jk}^i + T_{jk}^i = e^k_a e^a_{i,j}, \quad (25)$$

curvature tensor

$$S^i_{jkm} = 2\Delta^i_{j[m,k]} + 2\Delta^i_{s[k}\Delta^s_{|j|m]} = 0, \quad (26)$$

equals to zero. Then, substituting (25) into (26), we will get the relationship

$$S^i_{jkm} = R^i_{jkm} + 2\nabla_{[k}T^i_{|j|m]} + 2T^i_{c[k}T^c_{|j|m]} = R^i_{jkm} + P^i_{jkm} = 0. \quad (27)$$

Let us note that the connection of the geometry of absolute parallelism (25) has torsion

$$\Delta^i_{[jk]} = T^i_{[jk]} = -\Omega_{jk}^{\cdot i}, \quad (28)$$

which we will call *Ricci torsion*. Thus the geometry of absolute parallelism with the Riemannian curvature (23) and Ricci torsion (28) fits most of the implementation of Clifford's program for geometrization of physics.

4 Klein's "Erlangen Program" and Cartan's Structural Equations of the Geometry of Absolute Parallelism A_4

In 1872 F. Klein introduced the "Erlangen Program", which aimed to construct the basic geometrical relations for the geometry [8] specifying the group of motion of the space. This program had been consistently developed by many famous mathematicians with the major contribution made by Cartan. Cartan applied not a point manifold, which was used by Riemann to construct non-Euclidian geometry, but a manifold of the oriented points similar to Ricci. Cartan called an oriented point the "orthogonal moving reaper", which in motion created infinitesimal translations of the origin dx^i (in our case local group T_4) as well as infinitesimal turns of tetrad vectors de^i_a (local group $O(3.1)$). Using Cartan's method [9], we will obtain the following Cartan's structural equations of the geometry A_4 [2]

$$\nabla_{[k}e^a_{m]} - e^b_{[k}T^a_{|b|m]} = 0 \quad or \quad \nabla_{[a}e^i_{b]} = -\Omega_{ab}^{\cdot c}e^i_c, \quad (29)$$

$$R^a_{bkm} + 2\nabla_{[k}T^a_{|b|m]} + 2T^a_{c[k}T^c_{|b|m]} = 0 \quad or \quad R^a_{bkm} = 2e^a_i \nabla_{[k} \nabla_{m]} e^i_b, \quad (30)$$

which coincide with the Maurer-Cartan equations of the groups T_4 and $O(3.1)$ correspondingly.

In the equations (29) the Ricci torsion components $\Omega_{ab}^{\cdot c}$ represent the structural functions of the local group T_4 , satisfying first Jacobi's identity (or Bianchi's first identity)

$$\overset{*}{\nabla}_{[b} \Omega_{cd]}^{\cdot a} + 2\Omega_{[bc}^{\cdot f} \Omega_{d]f}^{\cdot a} = 0 \quad or \quad R^a_{[bcd]} = 0, \quad (31)$$

where $\overset{*}{\nabla}_b$ - the covariant derivative with respect to the connection (25). In the equations (30) the Riemannian tensor components R^a_{bkm} represent the structural functions of the local group $O(3.1)$, satisfying the second Jacobi's identity

$$\nabla_{[n} R^a_{|b|km]} + R^c_{b[km} T^a_{|c|n]} - T^c_{b[n} R^a_{|c|km]} = 0. \quad (32)$$

Considering that the structural equations (29) and (30) satisfy the conditions of integration (equations (31) and (32) correspondingly)[2], then the geometry of absolute parallelism happens to become the only geometry satisfying all the requirements of Klein's "Erlangen Program".

5 Inner Degrees of Freedom of an Oriented Point and Yang-Mills's Field Geometrization

The space of the events of mechanics of an oriented material point has a more complex structure, than the mechanics of a point. If the description of the dynamics of a material point in n -dimensional space requires n coordinates, then the description of the oriented material point in n -dimensional space requires $n(n+1)/2$ coordinates [10]. For example, in four dimensional space 10 coordinates define the oriented material point: four translational coordinates x, y, z, ct and six angular, where there are three space angles $\varphi_1, \varphi_2, \varphi_3$ and three space-time $\theta_1, \theta_2, \theta_3$. The anholonomic tetrad e^a_i represents the angular coordinates. The ten-dimensional manifold (four translational coordinates x^i and six "rotational coordinates e^i_a) of the geometry of absolute parallelism A_4 can be viewed as a vector fiber bundle with the coordinates of base x^i (external space), in each point of which there is a field of four orthonormal vectors e^i_c ($c=0,1,2,3$) [11] forming "inner" space. The translational group T_4 acts in the external space x_i (base) and the rotational group $O(3.1)$ acts in the "inner" space e^i_c (fiber) -. In the equations (29) and (30) the matrices e^a_i, T^a_{bk} and R^a_{bkm} are transformed in the rotational group $O(3.1)$ as follows

$$\begin{aligned} e^{a'}_i &= \Lambda_a^{a'} e^a_i \\ T^{a'}_{b'k} &= \Lambda_a^{a'} T^a_{bk} \Lambda^b_{b'} + \Lambda_a^{a'} \Lambda^a_{b',k}, \quad \Lambda_a^{a'} \in O(3.1), \\ R^{a'}_{b'km} &= \Lambda_a^{a'} R^a_{bkm} \Lambda^b_{b'}, \end{aligned} \quad (33)$$

while Ricci rotation coefficients T^a_{bk} perform as potentials of the gauge field R^a_{bkm} . Dropping the matrices indices, let us write the equations (30) and (32) in the form of geometrized Yang-Mills equations

$$R_{km} = 2\nabla_{[m} T_{k]} + [T_m, T_k], \quad (34)$$

$$\nabla_n \overset{*}{R}{}^{kn} + \overset{*}{R}{}^{kn} T_n - T_n \overset{*}{R}{}^{kn} = 0, \quad (35)$$

with the gauge group $O(3.1)$. We have introduced the notation for the dual Riemannian tensor $\overset{*}{R}{}_{ijkm} = \frac{1}{2} \varepsilon^{sp}{}_{km} R_{ijsp}$. Adding the structural equation of the translational group (29) to the geometrized Yang-Mills equations (34) and (35)

$$\nabla_{[k} e_{m]} - e_{[k} T_{m]} = 0, \quad (36)$$

we will get the extended system of geometrized Yang-Mills equations.

6 Equality of the Newman-Penrose Equations with Geometry A_4 Structural Equations

Clifford's program on the geometrization of physics started from Einstein's work, who

had shown that the relativistic gravitational fields and gravitational interactions can be described by the definite relationships of Riemannian geometry [12]. A. Einstein especially remarked, that a purely geometrical description of the gravitational fields could be given by Einstein's vacuum equations

$$R_{ik} = 0 \quad (37)$$

and these equations" represent the only rational fundamental case for the field theory that may pretend for strict approach. . ." [12]. Einstein was right and the Einstein's gravitational theory can be proven by the experiments, based upon the solutions of the Einstein's vacuum equations (37). In 1962 the mathematicians E.Newman and R. Penrose proposed a new method to search for the solutions of the Einstein's vacuum equations [13]. In the coordinates of the base x^i and with the accepted notations the basic equations of Newman-Penrose formalism can be viewed as follows

$$R^i_{jkm} + 2\nabla_{[k}T^i_{j|m]} + 2T^i_{s[k}T^s_{j|m]} = 0, \quad (2.7 NP)$$

$$\nabla_{[n}R_{ij|km]} + R^s_{j[km}T_{is|n]} - T^s_{j[n}R_{is|km]} = 0, \quad (2.9 NP)$$

$$\nabla_{[k}e^a_{j]} + T^i_{[kj]}e^a_i = 0. \quad (2.11 NP)$$

The numbers in the right part of the equations correspond to the numbers in Newman's and Penrose's work [13]. The comparison of these equations with the system (29)- (32) shows that Newman-Penrose formalism use the structural Cartan's equations of the geometry A_4 [2]. If we wish to obtain new solutions of Einstein's vacuum equations (37), there is no need to solve them now. It will be sufficient to find (or "construct") such a solution of the structural Cartan's equations of the geometry A_4 (29) and (30), which satisfy to $R_{ik} = 0$. Thus, such famous solutions as Schwarzschild [13], NUT [14] and Kerr [15] had been found for Einstein's vacuum equations.

7 Geometrization of Energy-Momentum Tensor in Einstein's Equations and Tensor Current in Yang-Mills Equations

After successful geometrization of gravitational interactions, A.Einstein introduced in theoretical physics the Unified Field Program that implied the geometrization of all other physical fields, which form of the material energy-momentum tensor in Einstein's equations

$$R_{jm} - \frac{1}{2}g_{jm}R = \frac{8\pi G}{c^4}T_{jm}. \quad (38)$$

In order to do so, Einstein used various generalizations of Riemannian geometry, including the geometry of absolute parallelism A_4 [16]. Although Einstein had actively corresponded with Cartan about the geometry of absolute parallelism [17], he was not aware at that time of Cartan's structural equations (29) and (30) of his geometry. Meanwhile the problem of

geometrization of the right part of Einstein's equations (38) can be solved with the help of Cartan's structural of the equations geometry A_4 . Let us write the equations (2.7 NP) as

$$C_{ijkm} + g_{i[k}R_{m]j} + g_{j[k}R_{m]i} + \frac{1}{3}Rg_{i[m}g_{k]j} + 2\nabla_{[k}T^i_{|j|m]} + 2T^i_{s[k}T^s_{|j|m]} = 0, \quad (39)$$

where C_{ijkm} – Weyl's tensor, R_{jm} –Ricci tensor, R - scalar curvature. These equations split into 10 equations [18]

$$R_{jm} - \frac{1}{2}g_{jm}R = \nu T_{jm}, \quad (40)$$

similar to Einstein's equations, but with geometrized right part, defined as

$$T_{jm} = -\frac{2}{\nu}\{(\nabla_{[i}T^i_{|j|m]} + T^i_{s[i}T^s_{|j|m]}) - \frac{1}{2}g_{jm}g^{pn}(\nabla_{[i}T^i_{|p|n]} + T^i_{s[i}T^s_{|p|n]})\} \quad (41)$$

and 10 equations

$$C_{ijkm} + 2\nabla_{[k}T^i_{|j|m]} + 2T^i_{s[k}T^s_{|j|m]} = -\nu J_{ijkm}, \quad (42)$$

similar to Yang-Mills equations, but with geometrized tensor current

$$J_{ijkm} = 2g_{[k(i}T_{j)m]} - \frac{1}{3}Tg_{i[m}g_{k]j}, \quad (43)$$

where T -trace of tensor (41). Certainly the equations (40) principally differ from the Einstein's equations (38), because they:

- a) represent the natural generalization of vacuum equations (37) and as well as the equations (37) do not contain any physical constant;
- b) are completely geometrized and describe the material fields through Ricci torsion (28);
- c) are self-complying with geometrized Yang-Mills equations (42) and " coordinate" equations (2.11NP).

For example, instead of Einstein's vacuum equations (37), from the equations (29), (30) we will get the system

$$\nabla_{[k}e^a_{j]} + T^i_{[kj]}e^a_i = 0, \quad (44)$$

$$C^i_{jkm} + 2\nabla_{[k}T^i_{|j|m]} + 2T^i_{s[k}T^s_{|j|m]} = 0. \quad (45)$$

E. Newman, R. Penrose and others have been finding the solution of this particular system for Einstein's vacuum. With the chosen coordinate system x^i , as a searched function it includes components of Weyl's tensor C^i_{jkm} , components of Ricci rotation coefficients T^i_{kj} as well as the components of tetrad e^a_j . For example, the solution with Schwarzschild's metric

$$ds^2 = \left(1 - \frac{2\Psi^0}{r}\right) c^2 dt^2 - \left(1 - \frac{2\Psi^0}{r}\right)^{-1} dr^2 - r^2(d\theta^2 + \sin^2\theta d\varphi^2),$$

in the coordinates $x_0 = ct$, r , $x^2 = \theta$ $x^3 = \varphi$ and in spinor presentation [13] it can be viewed for:

1. Components of Newman-Penrose symbols:

$$\sigma^i_{00} = (0, 1, 0, 0), \quad \sigma^i_{11} = (1, U, 0, 0), \quad \sigma^i_{01} = \rho(0, 0, P, iP),$$

$$\begin{aligned}\sigma_i^{00} &= (1, 0, 0, 0), & \sigma_i^{11} &= (-U, 1, 0, 0), & \sigma_i^{0i} &= -\frac{1}{2\rho P}(0, 0, 1, i), \\ U &= -1/2 + \Psi^0/r, & P &= (2)^{-1/2}(1 + \zeta\bar{\zeta}/4), & \zeta &= x^2 + ix^3, \\ & & & & \Psi^0 &= \text{const.}\end{aligned}$$

2. Spinor components of Ricci rotation coefficients:

$$\begin{aligned}\rho &= -1/r, & \alpha &= -\bar{\beta} = -\alpha^0/r, & \gamma &= \Psi^0/2r, \\ \mu &= -\varepsilon^0/r + 2\Psi^0/r^2, & \alpha^0 &= \zeta/4.\end{aligned}$$

3. Spinor components of Weyl's tensor:

$$\Psi = -\Psi^0/r^3.$$

Substituting the components of Ricci rotation coefficients of the given solution into the rotational metric (22), we will find:

$$d\tau^2 = -\frac{(\Psi^0)^2}{2r^4}dx_0^2 - \frac{2(\Psi^0 - r)}{r}d\theta^2 - \frac{2(\Psi^0 - r)\sin^2\theta}{r}d\varphi^2. \quad (46)$$

Besides, the received solution of new vacuum equations, has got physical sense, if we set

$$\Psi^0 = MG/c^2. \quad (47)$$

The principle difference of the equations (44) and (45) from Einstein's vacuum equations (37) is that, if Ricci torsion (consequently of Ricci rotation coefficients as well) in the equations (44) and (45) goes to zero, than we will get the flat space.

8 Motion Equations of Oriented Point. Physical Interpretation of the Ricci Rotation Coefficients

The motion equations of four-dimensional oriented point follows from the definition of connection of the geometry A_4 (25). Let us rewrite the relationship (25) as

$$\partial_k e^i_a + \Delta_{jk}^i e^j_a = 0,$$

or as

$$de^i_a + \Delta_{jk}^i e^j_a dx^k = 0.$$

Dividing this equation by ds , we will get the motion equations of the oriented point as

$$\frac{de^i_a}{ds} + \Delta_{jk}^i e^j_a \frac{dx^k}{ds} = 0 \quad (48)$$

or

$$\frac{de^i_a}{ds} + \Gamma_{jk}^i e^j_a \frac{dx^k}{ds} + T_{jk}^i e^j_a \frac{dx^k}{ds} = 0. \quad (49)$$

From 16 "rotational" equations (49), with the normalization condition (16), there remains 6 independent equations. These equations describe the change of the orientation of the

oriented point. It is possible to add 4 motion equations of the "origin" of oriented point, which represent the geodesic equations of the space A_4

$$\frac{d^2 x^i}{ds^2} + \Delta_{jk}^i \frac{dx^j}{ds} \frac{dx^k}{ds} = \frac{d^2 x^i}{ds^2} + \Gamma_{jk}^i \frac{dx^j}{ds} \frac{dx^k}{ds} + T_{jk}^i \frac{dx^j}{ds} \frac{dx^k}{ds} = 0. \quad (50)$$

From the relations (19) follows

$$T_{(jk)}^i = g^{im}(g_{js}\Omega_{mk}^{\cdot\cdot s} + g_{ks}\Omega_{mj}^{\cdot\cdot s}) = 2g^{im}\Omega_{m(jk)}, \quad (51)$$

thus the equations (50) will be written as

$$\frac{du^i}{ds} + \Gamma_{kj}^i u^j u^k + 2g^{im}\Omega_{m(jk)} u^j u^k = 0, \quad (52)$$

where we have noted $u^i = dx^i/ds$.

Let us remark, that:

- 1) the equations (52) could be obtained from variation principle [2];
- 2) the equations (52) followed from the equations of the oriented point (49), if we chose vector $e_i^{(0)} = dx^i/ds$.

If we multiply the equations (50) by mass m of the oriented point, then we will get the motion equations of its center of mass

$$m \frac{d^2 x^i}{ds^2} + m \Gamma_{jk}^i \frac{dx^j}{ds} \frac{dx^k}{ds} + m T_{jk}^i \frac{dx^j}{ds} \frac{dx^k}{ds} = 0. \quad (53)$$

In nonrelativistic approximation the equations (53) will be viewed as

$$m \frac{d^2 x^\alpha}{dt^2} = -mc^2 \Gamma_{00}^\alpha - mc^2 T_{00}^\alpha. \quad (54)$$

Applying the solution of vacuum equations (44) and (45) with Schwarzschild's metric, where the source function Ψ^0 is defined by the relation (47), we will obtain in quazi- Cartesian coordinates

$$F_G^\alpha = -mc^2 \Gamma_{00}^\alpha = m \frac{MG}{r^3} x^\alpha, \quad (55)$$

$$F_I^\alpha = -mc^2 T_{00}^\alpha = -m \frac{MG}{r^3} x^\alpha. \quad (56)$$

Evidently, the first of these forces F_G^α – Newtonian gravitational force. The force F_I^α is equal in its absolute value to the gravitational force F_G^α , but directed in the opposite side. We may naturally interpret it as an inertial force, which acts locally in the accelerated reference frame and compensates gravitational force, creating a weightless condition in free falling Einstein's lift. Correspondingly the Ricci rotation coefficients T_{jk}^i interpreted as intensity of the inertial field [19]. From the rotational metric (22) we will find the infinitesimal turn of the oriented point

$$d\chi^i_j = T_{jk}^i dx^k. \quad (57)$$

Dividing the right and left parts of these equations by ds , we will get a matrix of the four-dimensional angular velocity [2]

$$\Omega^i_j = T^i_{jk} \frac{dx^k}{ds} \quad (58)$$

with the property of symmetry $\Omega_{ik} = -\Omega_{ki}$. Let an oriented material point move now under the action of the inertial field T^i_{jk} only, then the motion equations (53) with the consideration of (58), could be presented as

$$m \frac{d^2 x^i}{ds^2} + m \Omega^i_j \frac{dx^j}{ds} = 0. \quad (59)$$

In non relativistic approximation from (59) we will have

$$m \frac{dv_\alpha}{dt} = -mc^2 \Omega_{\alpha 0} - 2mc^2 \Omega_{\alpha\beta} \frac{1}{c} \frac{dx^\beta}{dt}. \quad (60)$$

From another side , from nonrelativistic mechanics of the accelerated reference frames we will get the following equations, describing the motion of its origin [20]

$$m \frac{dv_\alpha}{dt} = m(-W_\alpha + 2\omega_{\alpha\beta} \frac{dx^\beta}{dt}), \quad \alpha, \beta = 1, 2, 3, \quad (61)$$

where

$$-mW_\alpha$$

- translational inertial force,

$$2m\omega_{\alpha\beta} \frac{dx^\beta}{dt}$$

- Coriolis force. Comparing the equations (60) and (61), we will find the matrix of four-dimensional angular velocity (matrix of the four-dimensional "classical spin") as

$$\Omega_{ij} = \frac{1}{c^2} \begin{pmatrix} 0 & -W_1 & -W_2 & -W_3 \\ W_1 & 0 & -c\omega_3 & c\omega_2 \\ W_2 & c\omega_3 & 0 & -c\omega_1 \\ W_3 & -c\omega_2 & c\omega_1 & 0 \end{pmatrix}. \quad (62)$$

We can see from this matrix that the four-dimensional rotation of the oriented material point is created by the inertial fields T^i_{jk} and vice versa - the rotation of matter originates Ricci torsion

$$\Omega_{jk}^{\cdot\cdot i} = -T^i_{[jk]} \quad (63)$$

of the space in geometry A_4 . The fields, defined by the spatial rotation, have been called torsion fields. Thus , the inertial field T^i_{jk} represents the torsion field, originated by the torsion of absolute parallelism geometry. The connection between rotation of matter and torsion (63) of A_4 geometry was outlined by Cartan in 1922 [21], although without a direct analytical reasoning. This fact created a stir in the research world. The reason was ,that a few years later Cartan introduced a torsion , based upon the point manifold. It differs from Ricci torsion (63), because it does not depend upon the angular variables. I could not find

any analytical proof of the connection of Cartan torsion (not Ricci torsion (63)) with real physical rotation.

9 Carmeli's Rotational Relativity. Spinor Structure of Penrose's Space of Events and Quantum Theory

For many years Einstein searched for the "intelligent generalization" of vacuum equations (37). He thought, that such generalization should provide "the key to a more perfect quantum theory" [22], and energy -momentum tensor should be geometrized and created by a field " of unknown nature " at that time [23]. If we consider that in Descartesian mechanics the equations (29) and (30) could become " an intelligent generalization" of vacuum equations (37), and the geometrized tensor of energy-momentum (41) provides the solution for geometrization of the material fields, then "the field of unknown nature" will be presented by inertial field T^i_{jk} . Defining the material density as

$$\rho = g^{jm} T_{jm} / c^2, \quad (64)$$

and applying energy-momentum tensor (41) we have the following

$$\rho = \frac{2g_{jm}}{\nu c^2} (\nabla_{[i} T^i_{|j|m]} + T^i_{s[i} T^s_{|j|m]}). \quad (65)$$

Suppose, that in the motion equations (53) the external forces

$$-m \Gamma^i_{jk} \frac{dx^j}{ds} \frac{dx^k}{ds}$$

equal to zero, then the center of masses of the oriented point is affected only by inertial forces

$$m \frac{d^2 x^i}{ds^2} + m T^i_{jk} \frac{dx^j}{ds} \frac{dx^k}{ds} = 0. \quad (66)$$

If inertial forces compensate each other, then we will get the following equation

$$m T^i_{jk} \frac{dx^j}{ds} \frac{dx^k}{ds} = 0,$$

solution of which gives us [2]

$$T_{ijk} = -T_{jik} = -T_{ikg} = -\Omega_{ijk}. \quad (67)$$

Meanwhile the origin of the reference frame, connected with the oriented material point, moves linearly and uniformly. From the relationships (67)) follows that the inertial field is skew-symmetrical in all three indices and coincides with Ricci torsion. Suppose, the origin of the accelerated reference frame, which satisfies the conditions (67), is connected with the reference frame, which does not change its vectors' orientation. Such a system will behave as an inertial reference frame and we will call it as *a local inertial frame of reference of the second kind*¹. The actual examples of this local inertial frame of the second

¹A local inertial frame of reference of the first kind is connected with free falling Einstein's lift.

kind are presented in fig. 1. For the inertial fields, satisfying the conditions of complete antisymmetry (67), the density of matter (65) will be viewed simply as

$$\rho = -\frac{1}{2\nu c^2} h_i h^i, \quad (68)$$

where the pseudo-vector h_i ("spin" of the oriented material point) connected with Ricci torsion as

$$\Omega^{ijk} = \varepsilon^{ijkm} h_m, \quad \Omega_{ijk} = \varepsilon_{ijkm} h^m \quad (69)$$

and ε_{ijkm} – the Levi-Chivita pseudotensor.

In the spinor basis and after the normalization of the fields h_i to unit, the density (68) *resembles the density of a mater in the quantum theory*. This result confirms again the intuitive vision of Einstein about the relativistic nature of "a perfect" quantum theory. Similar conclusions have been made by one of the Einstein's followers, Israeli physicist, Moshe Carmeli. He was the first to remark, that the Special Relativity Theory (Translational Relativity) may be extended by including the rotational Relativity (anholonomic rotational coordinates) [24], [25]. The original physical idea, applied by Carmeli, uses the known fact, that photons have not only a constant velocity of the translational motion c , but also a constant spin \hbar . That inspired M. Carmeli to introduce, additionally to Minkovsky metric,

$$ds^2 = dt^2 - c^{-2}[(dx^1)^2 + (dx^2)^2 + (dx^3)^2],$$

the rotational metric

$$d\tau^2 = dt^2 - \gamma^{-2}[(d\Theta^1)^2 + (d\Theta^2)^2 + (d\Theta^3)^2], \quad (70)$$

where γ - the factor, reflecting the limit of angular velocity of rotation , Θ^α ($\alpha = 1, 2, 3$) - anholonomic angular variables. Additionally to the principles of the Special Relativity Theory , Carmeli formulated two principles of Rotational Relativity:

1) the laws of physics are identical for all the references systems, rotating with the constant velocity relatively to each other;

2) the linear element (70) is invariant relative to the transformations of the anholonomic rotational coordinates [25].

If a rotating gyroscope is not affected by the external forces , then it will rotate with a constant velocity as long as possible. This mental experiment says, that in the Rotational Relativity Theory there is a "rotational" inertial principle (similar to Galilee's "translational" principle). It is a known fact, that rotation is an accelerated motion, thus the rotational relativity allows accelerated inertial motion. This is possible only in non-Euclidian space, such as the geometry of absolute parallelism. In other words, the further development of the Rotational Relativity Theory requires the change of Carmeli's metric (70) to the rotational relativistic metric (22). In this case will get organic synthesis of the translational and rotational relativity, providing the solutions for the program, introduced by Einstein. Carmeli's work [25] shows that the spin of the elementary particles may appear as a mediator, that connects relativity the theory with the quantum theory. Since the spinor represents the relativistic generalization of spin, then, as it was correctly remarked by G. Whiller [26], for the unification of relativity theory with the quantum theory we have to geometrize the spinor fields (for example, Dirac's spinor field). G. Whiller did not know that R. Penrose had began his brilliant works [27],[28], in which he had shown that it

was the spinors that served as a base for the classical geometry and it was them which defined the topological and geometrical properties of space-time, for example, its dimension and signature. In their work E.Newman and R. Penrose [13] had practically substituted in the structural equations of the geometry A_4 (29) and (30) the vectors e_i^a of the oriented point, of the Ricci rotation coefficients T^i_{jk} and the Riemann curvature by spin-tensor of the corresponding rank. In order to do so, it was necessary to apply spinor geometry A_4 as a differential manifold X_4 , in each point M of which together with the translational coordinates x_i ($i = 0, 1, 2, 3$) there had been introduced the complex spinor space \mathcal{C}^2 [28]. With the help of Carmeli's spinor 2×2 matrices [29]-[31] the equations (36), (40) and (42) (generalization of Einstein's vacuum equations) for the right matter (in theory there is right and left matter and antimatter [2]) represented as

$$\begin{aligned} \partial_{CD}\sigma^i_{AB} - \partial_{AB}\sigma^i_{CD} &= (T_{CD})_A{}^P \sigma^i_{PB} + \sigma^i_{AR}(T^+_{DC})^{\dot{R}}_{\dot{B}} - \\ &- (T_{AB})_C{}^P \sigma^i_{PD} - \sigma^i_{CR}(T^+_{BA})^{\dot{R}}_{\dot{D}}, \end{aligned} \quad (\overset{+}{A}{}^s)$$

$$2\Phi_{ABC\dot{D}} + \Lambda\varepsilon_{AB}\varepsilon_{\dot{C}\dot{D}} = \nu T_{A\dot{C}B\dot{D}}, \quad (\overset{+}{B}{}^{s+}.1)$$

$$\begin{aligned} C_{A\dot{B}C\dot{D}} - \partial_{CD}T_{A\dot{B}} + \partial_{A\dot{B}}T_{C\dot{D}} + (T_{CD})_A{}^F T_{F\dot{B}} + (T^+_{DC})^{\dot{F}}_{\dot{B}} T_{A\dot{F}} - \\ - (T_{A\dot{B}})_C{}^F T_{F\dot{D}} - (T^+_{BA})^{\dot{F}}_{\dot{D}} T_{C\dot{F}} - [T_{A\dot{B}}, T_{C\dot{D}}] = -\nu J_{A\dot{B}C\dot{D}}, \end{aligned} \quad (\overset{+}{B}{}^{s+}.2)$$

where the constant ν has got the notation $\nu = (8\pi G)/c^4 = \nu_g$ for the gravitational interactions or $\nu = (8\pi e)/m_0 c^4 = \nu_e$ for the electromagnetic interactions [2]. The equations ($\overset{+}{B}{}^{s+}.1$) are the spinor representation of the completely geometrized (including the tensor of energy impulse of matter) Einstein's equations, while the source $T_{A\dot{C}B\dot{D}}$ in general is defined through two-component spinors o_α, τ_β and their derivatives [32]. From another side, the equations ($\overset{+}{B}{}^{s+}.2$) represent completely geometrized Yang-Mills equations, where current $J_{A\dot{B}C\dot{D}}$ has been defined through two-component o_α, ι_β . The first structural equations ($\overset{+}{A}{}^s$) of geometry A_4 , noted through two-component spinor o_α, ι_β , satisfy the system of non linear spinor equations of the following type [2]

$$\begin{aligned} \nabla_{\beta\dot{\chi}} o_\alpha &= \gamma o_\alpha o_\beta \bar{o}_{\dot{\chi}} - \alpha o_\alpha o_\beta \bar{l}_{\dot{\chi}} - \\ &- \beta o_\alpha \iota_\beta \bar{o}_{\dot{\chi}} + \varepsilon o_\alpha \iota_\beta \bar{l}_{\dot{\chi}} - \tau \iota_\alpha o_\beta \bar{o}_{\dot{\chi}} + \\ &+ \rho \iota_\alpha o_\beta \bar{l}_{\dot{\chi}} + \sigma \iota_\alpha \iota_\beta \bar{o}_{\dot{\chi}} - \kappa \iota_\alpha \iota_\beta \bar{l}_{\dot{\chi}}, \\ \nabla_{\beta\dot{\chi}} \iota_\alpha &= \nu o_\alpha o_\beta \bar{o}_{\dot{\chi}} - \lambda o_\alpha o_\beta \bar{l}_{\dot{\chi}} - \\ &- \mu o_\alpha \iota_\beta \bar{o}_{\dot{\chi}} + \pi o_\alpha \iota_\beta \bar{l}_{\dot{\chi}} - \gamma \iota_\alpha o_\beta \bar{o}_{\dot{\chi}} + \\ &+ \alpha \iota_\alpha o_\beta \bar{l}_{\dot{\chi}} + \beta \iota_\alpha \iota_\beta \bar{o}_{\dot{\chi}} - \varepsilon \iota_\alpha \iota_\beta \bar{l}_{\dot{\chi}}, \end{aligned} \quad (71)$$

$$\alpha, \beta, \gamma \dots = 0, 1, \quad \dot{\chi}, \dot{\mu}, \dot{\nu} \dots = \dot{0}, \dot{1},$$

Due to the cubic nonlinearity (by two-component spinors) property of the equations (71) these equations have been called the generalized Heisenberg -Ivanenko nonlinear spinor equations [33], [34]. For the Schwarzschild solution of the spinor vacuum equations ($\overset{+}{A}{}^s$), ($\overset{+}{B}{}^{s+}.1$) and ($\overset{+}{B}{}^{s+}.2$) with the constant (47), the equations (71) will look as

$$\nabla_{\beta\dot{\chi}} o_\alpha = \frac{\Psi^0}{2r} o_\alpha o_\beta \bar{o}_{\dot{\chi}} + \frac{\bar{\alpha}^0}{r} o_\alpha o_\beta \bar{l}_{\dot{\chi}} - \frac{\alpha^0}{r} o_\alpha \iota_\beta \bar{o}_{\dot{\chi}} - \frac{1}{r} \iota_\alpha o_\beta \bar{l}_{\dot{\chi}},$$

$$\nabla_{\beta\dot{\chi}} l_{\alpha} = - \left(-\frac{1}{2r} + \frac{\Psi^0}{r^2} \right) o_{\alpha} l_{\beta} \bar{o}_{\dot{\chi}} - \frac{\Psi^0}{2r} l_{\alpha} o_{\beta} \bar{o}_{\dot{\chi}} - \frac{\bar{\alpha}^0}{r} l_{\alpha} o_{\beta} \bar{l}_{\dot{\chi}} - \frac{\alpha^0}{r} l_{\alpha} l_{\beta} \bar{o}_{\dot{\chi}},$$

$$\alpha, \beta \dots = 0, 1, \quad \dot{\gamma}, \dot{\chi} \dots = \dot{0}, \dot{1},$$

meanwhile $\Psi^0 = MG/c^2$ plays the role of the fundamental length. The generalized Heisenberg -Ivanenko equations - one more "clue" towards a more perfect quantum theory.

10 Inertial Mass in Descartesian Mechanics. Four-Dimensional Gyroscope

The inertial rest mass of an object in Descartesian mechanics is defined as

$$m_0 = \int \rho (-g)^{1/2} dV, \quad (72)$$

where

$$g = \det g_{jm}, \quad dV = dx^1 dx^2 dx^3,$$

and the density ρ is defined according to (65). The final expression of the inertial rest mass in Descartesian mechanics looks

$$m_0 = \frac{2}{\nu c^2} \int (-g)^{1/2} \left\{ g^{jm} \left(\nabla_{[i} T_{j|m]}^i + T_{s[i}^i T_{j|m]}^s \right) \right\} dV. \quad (73)$$

The relationships show that the inertial rest mass in Descartesian mechanics represents *the measure of the inertial field*. Since the inertial field T_{jk}^i originated by the rotation of the matter (according to E. Cartan) , then the inertial properties of the rest mass depend on the conditions of the rotation of the matter, forming the discussed system. For example, by changing the angular velocity of the separate mass parts of the system $m_0(t)$ according to a certain law, then we can create a "jet-like motion without rejecting the mass" according to the motion equations

$$m_0(t) \frac{d}{dt} (v_{\alpha}) = -v_{\alpha} \frac{d}{dt} m_0(t). \quad (74)$$

The mechanical device, where the center of mass moves according to the equations (74), has been called a four-dimensional gyroscope (4-D -gyroscope) (see fig. 1)

All the elements of the conventional 3-dimensional gyroscope rotate in the spatial angle ϕ in the planes, perpendicular to the axis of rotation. A 4-D gyroscope consists of three connected masses (see 1), two of which (masses m) rotate synchronously in different directions in the spatial angle $\phi(t)$ around axis O_1 , set on the central mass M . The central mass M itself oscillates along axis of symmetry x with the acceleration

$$W_x = \frac{dv_x(t)}{dt} = c(th\dot{\theta}_x) = c \frac{d[th \theta_x(t)]}{dt},$$

where θ – pseudo-Euclidean angle. That is why in Descartesian mechanics in terms of Lorentz local group the rotation of 4-D gyroscope is described by two matrixes. Particularly the spatial rotation of the small masses m is defined by matrix

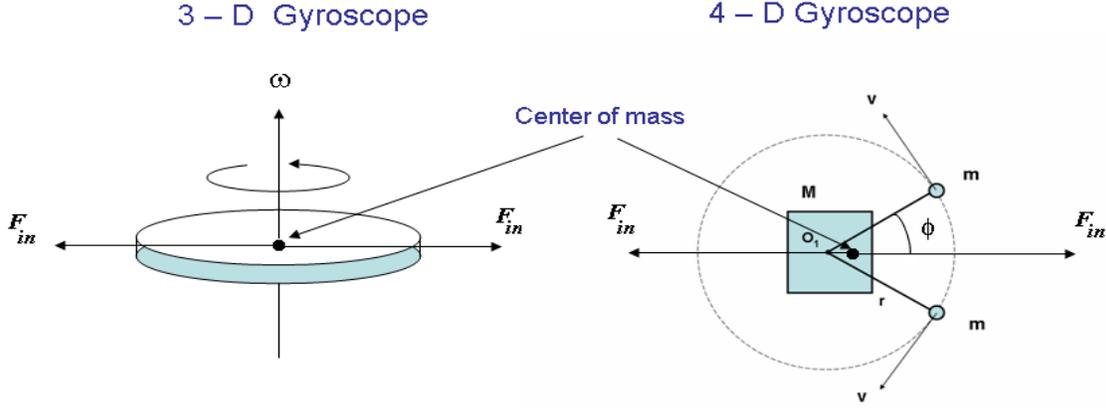


Figure 1: The local inertial reference frame of the second kind is connected with the center of masses of 3-D and 4-D free gyroscopes

$$R = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos \phi(t) & \sin \phi(t) & 0 \\ 0 & -\sin \phi(t) & \cos \phi(t) & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix},$$

and accelerated motion along axis x is described by the matrix

$$L = \begin{pmatrix} 1 & -th\theta(t)_x & 0 & 0 \\ -th\theta(t)_x & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}.$$

Thus, it becomes clear, why such a simple mechanical device has been called 4-D gyroscope. Lagrangian function T of 4-D gyroscope can be presented as

$$T = \frac{M + 2m}{2} (v_c^2 + k^2(1 - k^2 \sin^2 \phi)w^2) = \frac{M + 2m}{2} (v_c^2 + g'w^2), = \frac{M + 2m}{2} \dot{s}^2 \quad (75)$$

where

$$w = r\omega, \quad k^2 = 2m/(M + 2m), \quad v_c = v - k^2w \sin \phi, \quad g' = k^2(1 - k^2 \sin^2 \phi) = k^2g.$$

Here v_c - velocity of the center of masses, v - the velocity of the central mass M , $\omega = \dot{\phi}$ - angular velocity of the rotation of small masses, r - distance from O_1 to small masses m . Let us consider that the motion of the center of masses which is free from the action of the external forces of a 4-D gyroscope occurs according to the motion equations of Descartesian mechanics

$$\frac{d^2x^i}{ds^2} + \Delta^i_{jk} \frac{dx^j}{ds} \frac{dx^k}{ds} = 0, \quad (76)$$

$$i, j, k \dots = 1, 2$$

where

$$\begin{aligned} \Delta^i_{jk} &= \Gamma^i_{jk} + T^i_{jk} = e^i_a e^a_{j,k}. \\ ds^2 &= g_{ij} dx^i dx^j = \frac{2T}{M+2m} dt^2, \quad i, j = 1, 2, \\ g_{ij} &= \begin{pmatrix} 1 & 0 \\ 0 & g' \end{pmatrix} = \Lambda_{ab} e^a_i e^b_j, \\ \Lambda_{ab} &= \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}. \end{aligned} \tag{77}$$

The orthogonal diad e^a_i for the given metric tensor connected with the variables

$$v_c(t) = \cos \eta(t) \dot{s}, \quad \sqrt{g'} w(t) = \sin \eta(t) \dot{s}, \tag{78}$$

and can be viewed as

$$\begin{aligned} e^a_i(\eta(t)) &= \begin{pmatrix} \cos \eta & \sqrt{g'} \sin \eta \\ -\sin \eta & \sqrt{g'} \cos \eta \end{pmatrix}, \\ e^i_a(\eta(t)) &= \begin{pmatrix} \cos \eta & -\sin \eta \\ \frac{1}{\sqrt{g'}} \sin \eta & \frac{1}{\sqrt{g'}} \cos \eta \end{pmatrix}. \end{aligned}$$

After the corresponding calculations, the motion equations will become

$$\frac{dv_c}{dt} = \frac{2m}{M+2m} \Phi \omega, \tag{79}$$

$$\frac{d\omega}{dt} - k^2 \omega^2 \frac{\sin \phi \cos \phi}{1 - k^2 \sin^2 \phi} = -\frac{1}{rg} \Phi v_c, \tag{80}$$

where

$$\Phi(t) = -\frac{\sqrt{g'} d\eta}{k^2 dt} \tag{81}$$

- function, created by Ricci torsion. If this function goes to zero, then the equations (80) and (81) will coincide with the motion equations of 4-D gyroscope, which follow from Newtonian mechanics.

11 Space-Time Precession of a Free Four-Dimensional Gyroscope

The equations (80) and (81) could be presented as

$$\begin{aligned} \dot{v}_c^* &= k^2 \Phi^* w, \\ \dot{w} &= -\Phi^* v_c^*. \end{aligned}$$

where

$$v_c^* = v_c - v_0, \quad \Phi^* = \frac{\Phi}{\sqrt{g'}},$$

and $v_0 = \text{const}$ – the initial velocity of the center of mass.

Suppose, that

$$\Phi^* = \kappa_0 = \text{const},$$

then we will obtain the following special solutions for the motion equations

$$v_c(t) = v_0 \sin(k\kappa_0 t) + v_0 = v_0 (1 + \sin(k\kappa_0 t)), \quad (82)$$

$$\omega(t) = \frac{v_0}{\sqrt{g'}rk} \cos(k\kappa_0 t) + \frac{r\omega_0 \sqrt{g'(\phi_0)} - v_0/k}{r\sqrt{g'}},$$

which shows that, if the initial velocity of the center of mass differs from zero, there is the space-time precession of the 4-D gyroscope, which manifests in changing the velocity of the center of masses of the 4-D gyroscope, when it is free from the action of the external forces.

12 The Control of Metric of Local Space

If the 4-D gyroscope is affected by the external force F_x or the external force momentum L , then its motion equations (79) and (80) are viewed as

$$\frac{dv}{dt} - B \frac{d}{dt}(\omega \sin \phi) = \frac{F_x}{M + 2m} + B\Phi\omega, \quad (83)$$

$$r \frac{d\omega}{dt} - \frac{dv}{dt} \sin \phi = \frac{L}{2mr} - \Phi v. \quad (84)$$

Multiplying the first of those equations by $(M + 2m)v$, and the second one by $2mr\omega$, then adding them, we will get the law of changes of the total energy of the system

$$\frac{d}{dt} \left(\frac{1}{2}(M + 2m)v^2 + mr^2\omega^2 - 2mrv\omega \sin \phi \right) = F_x v + L\omega. \quad (85)$$

We can see from that law, that the torsion force (or non compensated inertial force)

$$F_i = (M + 2m)B\Phi\omega$$

and the torsion angular momentum (or non compensated inertial force angular momentum)

$$L_i = 2mrv\Phi$$

do not change the energy of the system, although actively participate in the energy distribution between translational and rotational motions of the 4-D gyroscope. Multiplying the equation (83) by $\sin \phi$ and adding it to the equation (84), we will find

$$r \frac{d\omega}{dt} - B \frac{d}{dt}(\omega \sin \phi) = \frac{F_x \sin \phi}{M + 2m} + Nr - \Phi(v - B\omega \sin \phi) \quad N = \frac{L}{2mr^2}. \quad (86)$$

Substituting in this equation and in the equation (83)

$$v_c = v - B\omega \sin\phi,$$

we get

$$\frac{dv_c}{dt} = \frac{F_x}{M + 2m} + B\Phi\omega, \quad (87)$$

$$r \frac{d\omega}{dt} g - B(\omega^2 \sin\phi \cos\phi) = \frac{F_x \sin\phi}{M + 2m} + Nr - \Phi v_c, \quad (88)$$

where

$$g = 1 - k^2 \sin^2\phi.$$

Let us introduce the notation

$$\psi = \frac{\Phi}{g}, \quad w = gr\omega$$

and rewrite (87) and (88) as

$$\frac{dv_c}{dt} = \frac{F_x}{M + 2m} + k^2\psi w, \quad (89)$$

$$\frac{dw}{dt} = \frac{F_x \sin\phi}{(M + 2m)g} + \frac{Nr}{g} - \psi v_c. \quad (90)$$

Multiplying the first of them by w , and the second one by $-v_c$, then adding them, we will get

$$w \frac{dv_c}{dt} - v_c \frac{dw}{dt} = -\frac{F_x \sin\phi v_c}{(M + 2m)g} - \frac{Nr v_c}{g} + \frac{F_x w}{M + 2m} + \psi(k^2 w^2 + v_c).$$

Since the total energy of the 4-D gyroscope equals

$$T = \frac{1}{2}(k^2 w^2 + v_c^2),$$

then we obtain specific commutator

$$w \frac{dv_c}{dt} - v_c \frac{dw}{dt} = -\left\{ \frac{F_x \sin\phi}{(M + 2m)g} + \frac{Nr}{g} \right\} v_c + \frac{F_x w}{M + 2m} + \frac{2T\psi}{M + 2m}.$$

Multiplying the equations (89) by $(M + 2m)v_c$, and the equation (90) by $2mw$, then adding them, we will get the law of changes of the total energy of 4-D gyroscope after the action of the external forces and momentums

$$\frac{d}{dt} \left\{ \frac{1}{2}(M + 2m)v_c^2 + (1 - k^2 \sin^2\phi)mr^2\omega^2 \right\} = F_x v_c + BF_x \omega \sin\phi + L\omega \quad (91)$$

or

$$\frac{d}{dt} T(t) = F_x v_c + BF_x \omega \sin\phi + L\omega. \quad (92)$$

Since

$$T(t) = \frac{M + 2m}{2} \dot{s}(t)^2 = \int_0^t (F_x v_c + BF_x \omega \sin\phi + L\omega) d\tau,$$

then

$$\dot{s} = \sqrt{\frac{2T}{M+2m}} \neq \text{const}$$

and local metric ds^2 becomes dependable from the action of external forces and angular momentums

$$ds^2(t) = \frac{2T(t)}{M+2m} dt^2 = \frac{2}{M+2m} \left\{ \int_0^t (F_x v_c + B F_x \omega \sin \phi + L \omega) d\tau \right\} dt^2 \neq \text{inv.} \quad (93)$$

The formula (93) is remarkable, because with the absence of the action of the external forces F_x it is possible to change local space metric with the help of the angular momentum L . In practice we can do it inside the sealed body by adjusting a special device, called motor-break [35], on the 4-D gyroscope. Changing and controlling local angular momentum L , we change the local space metric and, naturally, the velocity of the center of masses of the system.

13 Control of Ricci Torsion and Riemann Curvature of Local Space

Descartesian mechanics requires four-dimensional coordinate space for the description of 4-D gyroscope, even for velocity much less than the speed of light

$$x_0 = ct, \quad x_1 = x, \quad x_2 = y, \quad x_3 = z.$$

It follows from the fact that translational acceleration in Descartesian mechanics is deduced to the rotation in the space-time planes, for example, as in our case, according to the formula

$$W_x = \frac{dv_x(t)}{dt} = c(th\dot{\theta}_x) = c \frac{d[th \theta_x(t)]}{dt}.$$

That is why for a more consistent description of 4-D gyroscope we have to apply the coordinates

$$x_0 = ct, \quad x_1 = x_c, \quad x_2 = r\phi.$$

We will select the metric tensor of the following type

$$g_{ij} = \begin{pmatrix} 0 & 1 - 2k^2 r^2 U(\phi)/c^2 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -k^2(1 - k^2 \sin^2 \phi) \end{pmatrix}, \quad (94)$$

where the "potential"

$$U(\phi) = \int_{\phi_0}^{\phi} N d\phi \quad (95)$$

is created by angular acceleration

$$N = \frac{L}{2mr^2}.$$

The motion equations of 4-D gyroscope will be written as

$$\frac{d^2 x^i}{ds^2} + \Gamma_{jk}^i \frac{dx^j}{ds} \frac{dx^k}{ds} + T_{jk}^i \frac{dx^j}{ds} \frac{dx^k}{ds} = 0 \quad (96)$$

$$i, j, k = 0, 1, 2$$

Applying the metric tensor (94), we will find the following components of Christoffel's symbols, which differ from zero

$$\Gamma_{02}^0 = \Gamma_{20}^0 = -\frac{k^2 r N}{c^2 - 2k^2 r^2 \int N d\phi}, \quad \Gamma_{00}^2 = -\frac{r N}{c^2(1 - k^2 \sin^2 \phi)}, \quad (97)$$

$$\Gamma_{22}^2 = -\frac{k^2 \sin \phi \cos \phi}{r(1 - k^2 \sin^2 \phi)}.$$

Since the Ricci torsion in the Cartan's structural equations of the geometry A_4 does not depend on metric, then we chose components of Ricci torsion Ω_{ijk} so, that in non relativistic limit, the equations (96) for non free 4-D gyroscope coincide with the motion equations (83) and (84)

$$\Omega_{02}^1 = -\Omega_{20}^1 = k^2 \Phi / 2c, \quad \Omega_{01}^2 = -\Omega_{10}^2 = -\frac{\Phi}{2c(1 - k^2 \sin^2 \phi)}. \quad (98)$$

Ricci rotation coefficients, accordingly, will be viewed as

$$T_{20}^1 = -k^2 \Phi / c, \quad T_{10}^2 = \frac{\Phi}{c(1 - k^2 \sin^2 \phi)}. \quad (99)$$

Now in order to calculate the controllable Ricci torsion fields (functions $\Phi(t)$) we will use the Cartan's structural equations

$$\nabla_{[k} e^a_{j]} + T^i_{[kj]} e^a_i = 0, \quad (100)$$

$$S^i_{jkm} = R^i_{jkm} + P^i_{jkm} = 0, \quad (101)$$

$$i, j, k, \dots = 0, 1, 2, \quad a, b, c, \dots = 0, 1, 2,$$

where

$$P^i_{jkm} = 2\nabla_{[k} T^i_{j|m]} + 2T^i_{s[k} T^s_{j|m]}. \quad (102)$$

Let us remember, that the equations (101) interconnect Riemann's curvature and the Ricci torsion of absolute parallelism geometry. Let us use it. We form the analogue of Ricci tensor for tensor S^i_{jkm} .

$$S_{jm} = S^i_{jim} = R^i_{jim} + P^i_{jim} = R_{jm} + P_{jm} = 0. \quad (103)$$

Correspondingly, for the scalar curvature of this tensor we will have

$$S = g^{jm} S_{jm} = g^{jm} (R_{jm} + P_{jm}) = R + P = 0. \quad (104)$$

Helped by the relations (97), we find

$$R_{00} = -\frac{r^2 k^2 U_\phi^2}{c^2 g (c^2 - 2k^2 r^2 U)} - \frac{k^2 U_\phi \sin \phi \cos \phi}{c^2 g^2} - \frac{U_{\phi\phi}}{c^2 g}, \quad (105)$$

$$R_{22} = -\frac{k^2 c^2 g}{c^2 - 2k^2 r^2 U} R_{00},$$

$$R = g^{jm} R_{jm} = \frac{2c^2}{c^2 - 2k^2 r^2 U} R_{00}. \quad (106)$$

Substituting in (104) scalar Riemann curvature (106) and calculated with (99) P , get

$$\Phi = 2\sqrt{\frac{N \sin \phi \cos \phi}{1 - k^2 \sin^2 \phi} + \frac{N_\phi}{k^2}}. \quad (107)$$

Substitute this relation in the motion equation

$$\frac{dv_c}{dt} = r k^2 \Phi \omega,$$

we will find the following expression for uncompensated inertial force, acting on the center of masses of the 4-D gyroscope

$$F_{in} = 2(M + 2m)B\omega\sqrt{\frac{N \sin \phi \cos \phi}{1 - k^2 \sin^2 \phi} + \frac{N_\phi}{k^2}}. \quad (108)$$

This force is created by the local torsion, which, in its turn, creates its local Riemann curvature and, thus, causes the change of the velocity of the center of masses.

14 Experimental Investigations of Space-Time Precession of Four-Dimensional Gyroscope

For the experimental research of the 4-D gyroscope mechanics, its space -time precession, we created 11 models of the 4-D gyroscopes with the mechanical and electrical motor-breaks. Some of them have been operated by the computer software. We constructed the experimental bench-stand, consisting of the horizontal surface, the measuring system to register the translational coordinate $x(t)$ ($\Delta x = \pm 0.5mm$) and angular coordinates $\phi(t)$ ($\Delta \phi \pm 0.5^\circ$). The special software allowed us to calculate the linear and angular velocities in real time. The corresponding graphs have been monitored and observed during the experiments. We have researched the following:

- 1) space-time precession of the 4-D gyroscope,
- 2) absolute elastic external collision of the gyroscope's body against the wall, which allowed us to observe:
 - a) transformation of the translational inertia into rotational;
 - b) transformation of the rotational inertia into translational;
 - c) multiple impacts of the 4-D gyroscope;
- 3) singular internal collisions of the 4-D gyroscope (on the cart and while suspended);

- 4) multiple internal collisions of the 4-D gyroscope (on the cart and while suspended);
- 5) changes of the direction of the 4-D gyroscope's motion without changes of the direction of the rotation of its small masses m .

These experiments demonstrated that the motion of the center of masses of a 4-D gyroscope cannot be explained by Newtonian mechanics. The controllable operation of the motion of its center mass is explained by the space-time precession that is understandable from the point of view of Cartesian mechanics. However, perhaps, it is the first attempt of the scientific foundation of new mechanics and more detailed investigations are required.

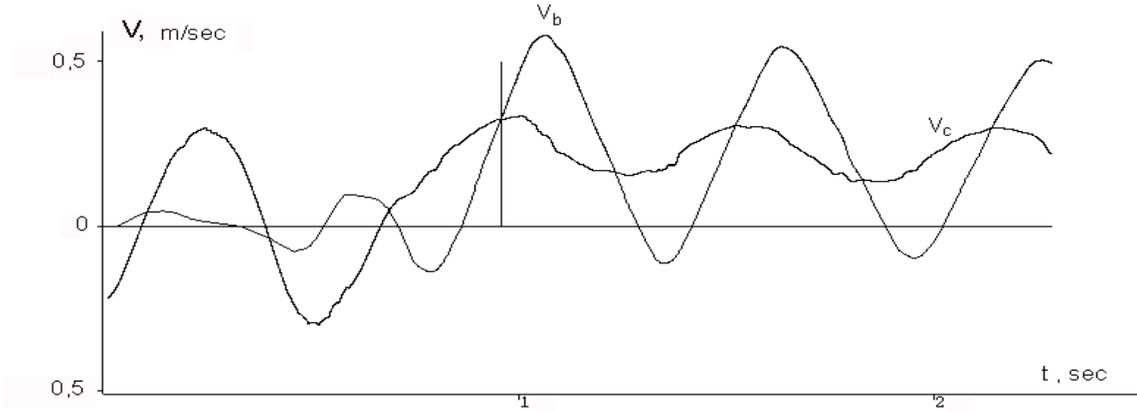


Figure 2: To the left from the vertical line, 4-D free gyroscope is affected by the external force, which creates space-time precession; after the line the 4-D gyroscope is free; v_b - body's velocity, v_c - velocity of the center of mass

Fig. 2 presents the typical graph of space -time precession of 4-D gyroscope, where v_b - body's velocity, v_c - the velocity of the center of mass. The absolute elastic impact implies the absolute elastic collision of its body. Otherwise the outsider observes an absolute elastic collision of the "black box"- the 4-D gyroscope - with the wall. With the condition that during the moment of brief impact ($t=0.01$ sec) the following conservation laws are fulfilled: total energy $T = T' = const$, the translational momentum of its body $P = (M + 2m)v' = (M + 2m)v = const$, and the angle $\phi' = \phi = const$, we will obtain the following relations [37]

$$P'_c = -P_c(1 - 2k^2 \sin^2 \phi) + 2K(1 - k^2 \sin^2 \phi), \quad (109)$$

$$K' = K(1 - 2k^2 \sin^2 \phi) + 2P_c k^2 \sin^2 \phi, \quad (110)$$

where

$$K = -2mr\omega \sin \phi$$

- rotational momentum. The relation (109),(110) presents 4-D gyroscope. As we can see, it generalizes the known momentum conservation law $P'_c = P_c$ for the absolute elastic collision of the solid body against the wall. It happens because the torsion forces of inertia, acting inside the 4-D gyroscope, provide the re- distribution between translational and rotational inertia after the action of velocity the external forces.

14.1 Absolute Elastic Collision, Demonstrating Transition of Translational Inertia into Rotational Inertia

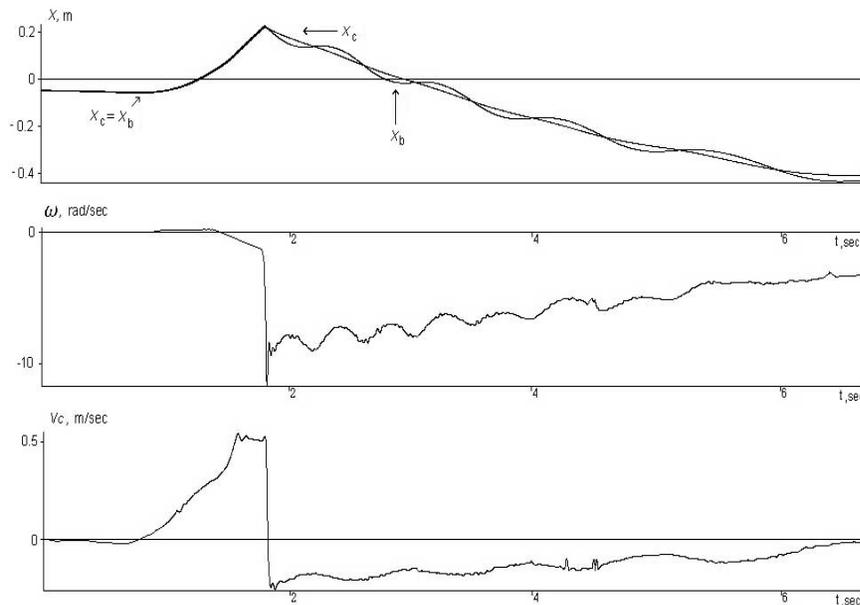


Figure 3: Absolute elastic collision of 4-D gyroscope, which transforms translational inertia into rotational inertia (x_c and x_b) - coordinates of the center of masses and the body accordingly

For the demonstration of this effect it is necessary to position the small masses under a certain angle towards the direction of motion. The major effect can be reached, when the angle makes 90° or 270° relative to the direction of motion. Afterwards we have to (slowly accelerating the gyroscope) direct it towards the wall. Slow acceleration will not permit the small masses to overcome the inner forces of friction and they will not start rotating before the collision. After the impact the small masses acquire angular rotational velocity, because the part of the translational inertia is transformed into rotational, however the velocity of the center of masses is not decreasing. (see Fig. 3) In Fig. 3 the upper graph depicts the coordinates of the central body x and the center of masses of the system x_c . Before the impact the curves coincide, after the impact the curve x oscillates around curve x_c . The next graph presents the angular velocity of masses rotation ω . The graph shows, that before the collision it equals to zero (within the limit of some measuring errors), and after the impact it changes up to a magnitude of 10 rad/sec. Hence, before the collision the system had translational inertia, and after the collision a part of translational inertia transformed into rotational inertia.

Below we can see the graph of changes of the velocity of the center of masses v_c . This velocity, before the collision equaled 50 cm/sec, and after the impact it became -25 cm/sec, i.e. double change in absolute value, that exceeds any potential experimental errors. With the corrections of the experimental errors, the curve ω and v_c described by the formulas, obtained earlier (109), (110). Hence, the total energy of the system is constant during the collision, and the change of the velocity of the center of masses after the impact in its

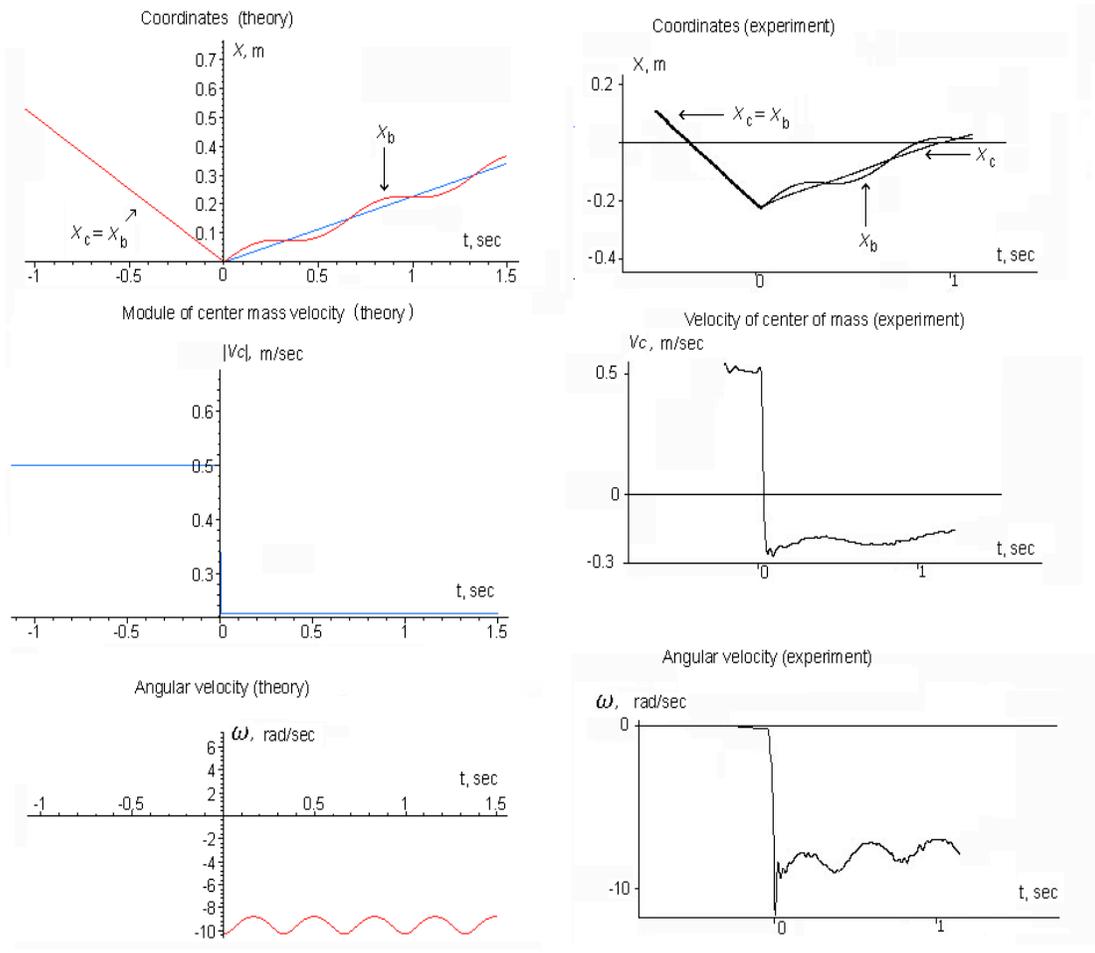


Figure 4: Comparison of the theoretical and experimental data on absolute elastic collision of 4-D gyroscope, which transforms translational inertia into rotational inertia

absolute value is explained by the transition of the part of the translational energy into inner rotational energy. Fig 4 presents the basic theoretical and experimental graphs of absolute elastic collision of the 4-D gyroscope with the transition of the translational inertia into rotational. After comparisons of the theoretical and experimental graphs we can assess that, within the limit of experimental errors, the theory describes the experiments correctly. The major part of the observed deviation of the experimental data from the theoretical forecast is explained by the absence of accounts of the friction forces, acting inside and outside of the 4-D gyroscope.

14.2 Absolute Elastic Collision, Demonstrating Transition of Rotational Inertia in Translational Inertia

Suppose that after the collision, the rotation of the small masses stopped ($K' = 0$), then their equations (110) look

$$K(1 - 2k^2 \sin^2 \phi) = -2P_c k^2 \sin^2 \phi.$$

Substituting this correlation in equation (109), we will get

$$P'_c = -P_c(1 + 2k^2 \sin^2 \phi).$$

From that equation we can read that during the transformation of the rotational inertia into translational, the absolute value of the velocity of the center of masses of the system increased. In this case, before directing the gyroscope towards the wall, it is required to

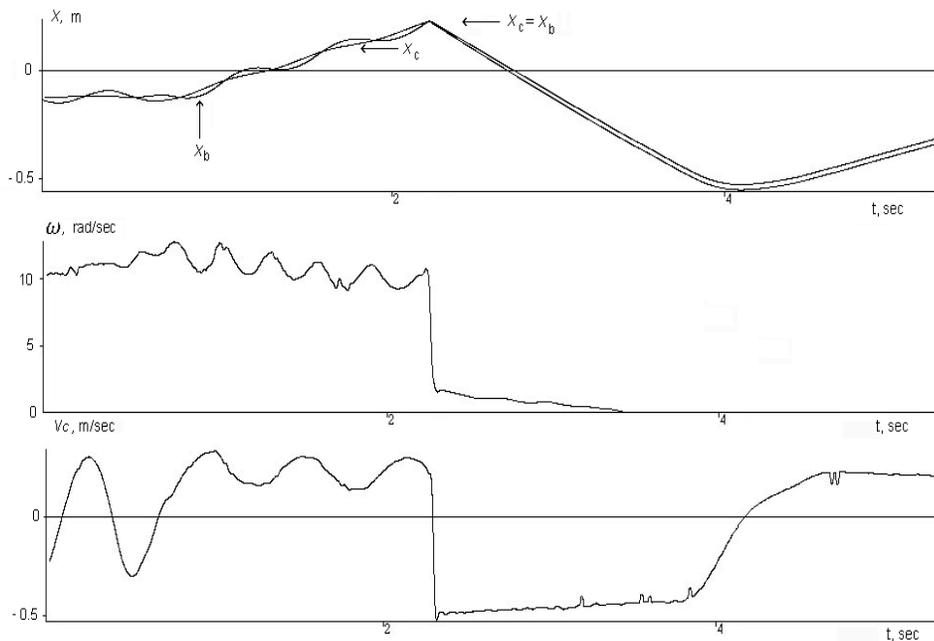


Figure 5: Absolute elastic collision of 4-D gyroscope, which transforms rotational inertia into translational inertia

initiate the rotation of the small masses. The best way is to perform it in a resonance way, i.e. vibrating the central body along axis x . After the masses begin rotating, the gyroscope should get some velocity towards the wall to perform the collision. Performing this experiment several times, one has to achieve the situation, when after the impact the angular velocity has to go to zero. Fig 5 demonstrates the graphs for this case. Before the collision the angular velocity ω was about 11 rad/sec. After the impact it equaled 2 rad/sec. The decrease of the angular velocity of the small masses rotation lead to the increase of the velocity of the center of masses v_c in absolute value. We can observe from the graph the change of the velocity v_c from 20 cm/sec up to -54 cm/sec, i.e. more than two times in absolute value.

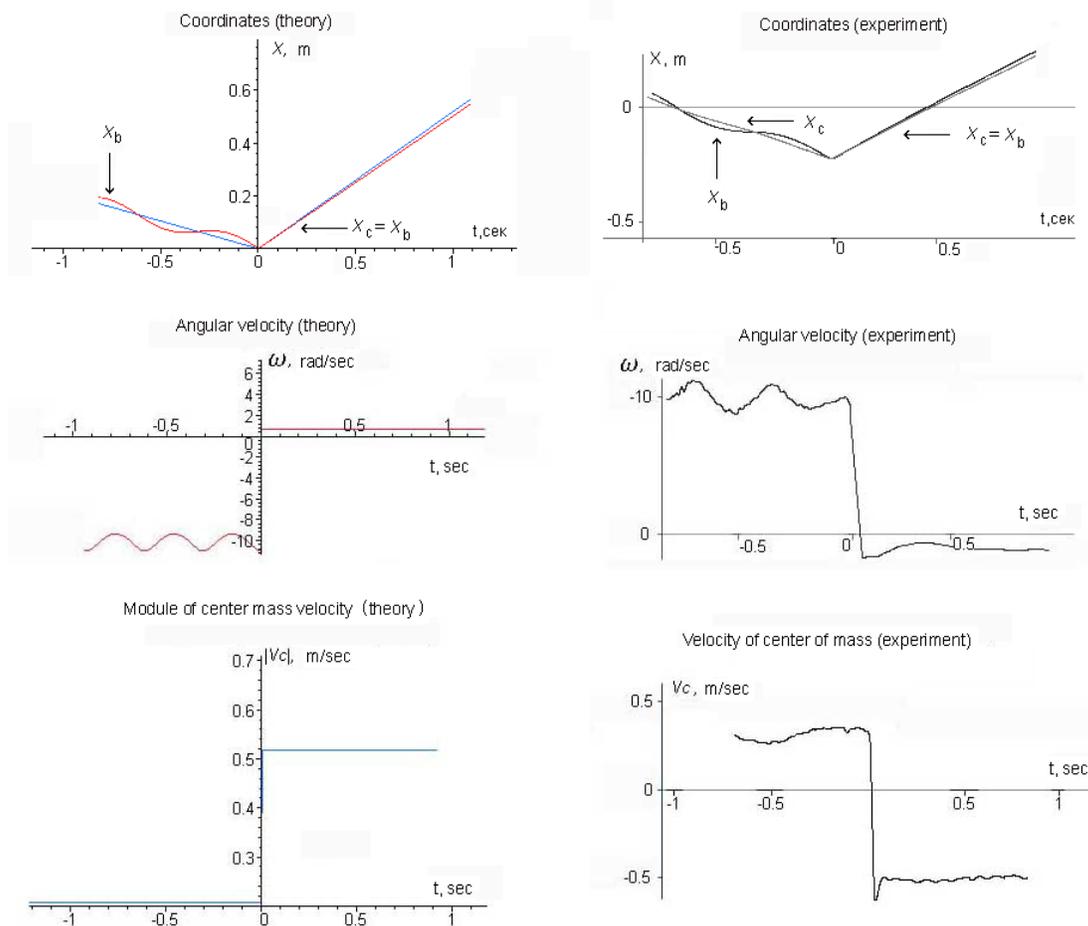


Figure 6: Comparison of the theoretical and experimental data on absolute elastic collision of 4-D gyroscope , which transforms rotational inertia into translational inertia

Fig 6 presents the theoretical and experimental graphs, depicting the transformation of the rotational inertia into translational inertia. The comparison of the graphs shows that the theory describes the experimental data well, implying only some errors during the experiments.

14.3 Experimental Investigations of Multiple Inner Impacts

We can cause the space-time precession of a 4-D gyroscope with the help of a special device, called motor-break. This device creates a sharp change of the angular velocity of the rotation of the small masses m in certain angular segments. That is why we will apply the expression *inner impact* for the big accelerations N , artificially created inside the 4-D gyroscope. According to the formula (108) the center of masses will be affected by the efficient force, changing its acceleration.



Figure 7: 4-D gyroscope with the electric motor

In order to achieve a more efficient motion of the 4-D gyroscope, due to organization of the inner impacts, we have developed a model, where the electric motor had been used as a power source.

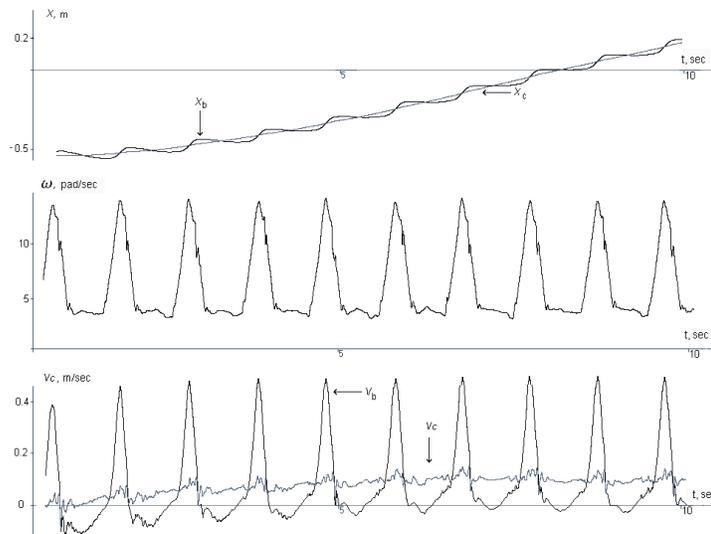


Figure 8: The experimental data of the multiple inner impacts of 4-D gyroscope with the electric motor; v_b - body's velocity, v_c - velocity of the center of mass

Increase and decrease of the angular velocity of the rotation happened in the electro-

magnetic way with the help of sensors that turned the motor on or slowed it down at the required time and required segments. The simplest model of such a device is presented in fig. 7. The experimental graphs of the motion of such a model are depicted in fig.8. For an observer the body of the 4-D gyroscope moves with the average velocity about 10 cm/sec. Meanwhile, during one cycle the body retreats 2 cm backwards and moves 12 cm forward.

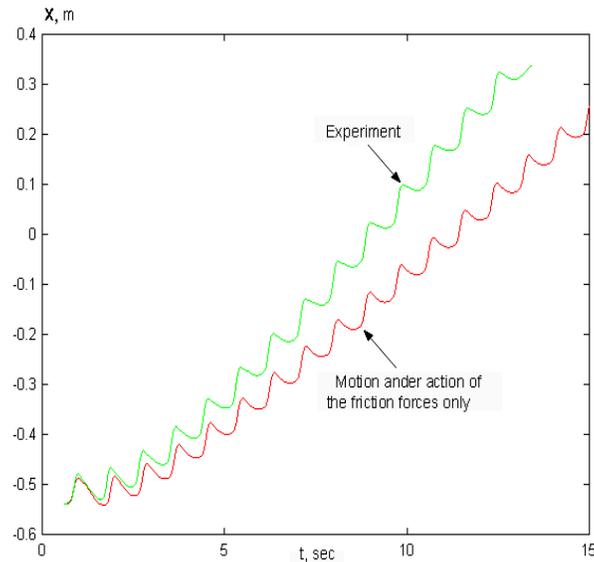


Figure 9: The differences between the theoretical and experimental curves, demonstrating the motion of the gyroscope, are born by friction forces only

This retreat caused some observers to think that during the motion of the wheels backwards the friction forces between the wheels and surface of motion moved the center of masses forward. In order to investigate the role of the friction forces we performed the special research of the friction forces effects. For this purpose, the model (with the rotating masses) was launched with some acceleration along the horizontal surface. Then the graph on deceleration motion has been registered, due to the friction forces, and the contribution of the friction forces has been calculated. Corresponding experimental and theoretical graphs for body's coordinates are in fig. 9. It is easy to notice that the experimental and theoretical graphs do not coincide. That means that the motion of the gyroscope could not be described by friction forces alone. The cause of the motion is connected with the inner impacts, appearing with the sharp change of the angular velocity. Theoretically it could be described with the help of Frenet's motion equations (not Newtonian).

14.4 The Model No.3 with Computerized Motion Control

Since the character of motion is fully defined by the law of the change of the frequency of the rotation of small masses, then it should be a good idea to operate it via computer. Moreover, if we wish to exclude the influence of the friction forces on the motion of the center of masses of the system forward, it is required to operate the motion of the

gyroscope body and, consequently, of its supporting wheels forward only. In this case the friction forces will always obstruct the motion of the center of masses forward, slowing down its motion.

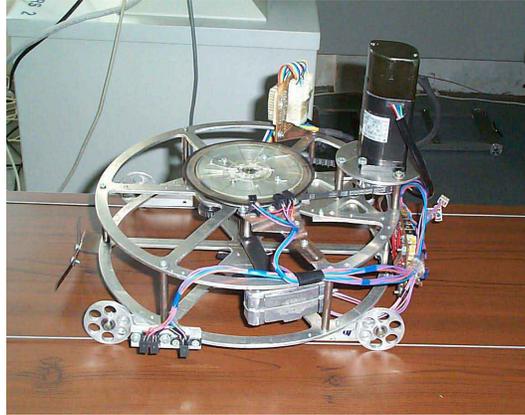


Figure 10: 4-D gyroscope with the computerized motion control

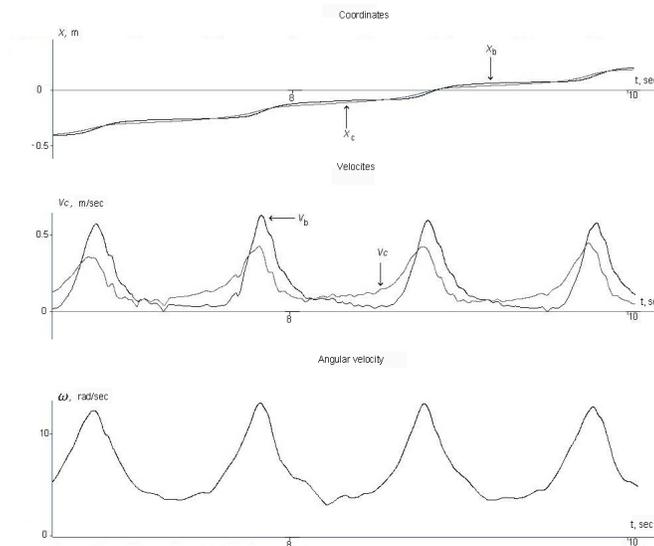


Figure 11: The experimental graphs of the 4-D gyroscope with computerized motion control; v_b - body's velocity, v_c - velocity of the center of mass

Fig. 10 presents 4-D gyroscope with servomotor (motor with feed back). The operation of this motor is performed via computer and special software. The program allows us to accelerate and slow down the rotation of small masses in the required segments. The graph of motion velocities (fig.11) for the body and center of masses shows that the body moves only forward. Accordingly, the wheels, supporting it, move only forward, while the friction forces, between the wheels and surface of motion, work against the motion and could not cause the motion forward.

Conclusion

The Theory of Elementary Particles is a leading edge of modern physics. At present, this theory is being developed by the inductive method, which is based upon the experimental work. A. Einstein thought that it was not possible to construct a complex theory inductively, because such a theory should, all the time, be "adjusted to observable events and thus might lead to enormous data base" [35]. That is why A. Einstein proposed to create complex physical theories by the deductive method, applying most general principles. It was a deductive method, which we have used to create Cartesian mechanics - the Fourth Generalization of Newtonian mechanics. Such generalization has become possible with regards that Cartesian mechanics that has been based upon the following:

- 1) Clifford-Einstein proposal for geometrization of physics (Unified Field Theory).
- 2) Klein's "Erlangen Program".
- 3) Cartan's idea about the connection of the torsion of space with physical rotation.
- 4) Einstein's idea about geometric nature of material fields.
- 5) Carmeli's idea about unification of the translational and rotational relativity.
- 6) Penrose's idea about similar transformational law for translations and rotations.
- 7) G. Whiller's idea about geometric nature of spinor fields.
- 8) Cartesian idea about rotational nature of any motion.

For the first time in physics the equations of Cartesian mechanics were applied by Newman and Penrose only as a method for finding the new solutions of Einstein's vacuum equations and not as independent physical equations. In 1988 the author proposed the equations of N-P formalism as equations of Physical Vacuum [36] (They are the equations of Cartesian mechanics as well). The principle of Universal Relativity laid the foundation for the equations of Physical Vacuum. This principle states that all physical fields and interactions have the relative nature. At the present stage of modern development in physics Einstein's program for creating the Universal Field Theory grew into the Theory of Physical Vacuum. The idea is very simple - if we know how Physical Vacuum (where the elementary particles are born) is arranged, we will know how these elementary particles are constructed, because we need the equations of the Unified Field Theory to describe their interactions. In this article we have adhered to the experimental verifications for some of the conclusions of new mechanics, using the known phenomenon [38], where the main role belongs to fields and forces of inertia - one of the great physical enigma from Newton's times. This is the Cartesian mechanics, which allows us to create the theoretical foundation for the experiments that Newton's mechanics could not explain; which demonstrate "jet-like motion without rejection of mass" [37]. The simplest model of the mechanical Propulsion System, which propels in space in jet-like motion, although without rejection of mass, had been created by a talented Russian engineer Vladimir Tolchin [38]. We have continued the experiments with Tolchin's mechanical devices and discovered that they deviated from Newton mechanics, when the center of mass had been affected by uncompensated forces of inertia, causing the phenomenon of space-time precession. We have observed that the phenomenon of space-time precession of four-dimensional gyroscope allows us to control its inertial mass. In the near future it will allow the creation of the Universal Propulsion System, which will be able to move in all the media: on earth, on water, under water, in air and in space. The 4-D Engine, with a hermetically sealed body, using space-time precession, will have quite a number of advantages and benefits, compared

to any other engine: it will be ecologically clean, economic and universal. It should gradually replace the existing engines in many branches of contemporary technologies.

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